Magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface

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ARTICLE INFO

ABSTRACT

Steady two-dimensional stagnation-point flow of an electrically conducting power-law fluid over a stretching surface is investigated when the surface is stretched in its own plane with a velocity proportional to the distance from the stagnation-point. We have discussed the uniqueness of the solution except when the ratio of free stream velocity and stretching velocity is equal to 1. The effect of magnetic field on the flow characteristic is explored numerically and it is concluded that the velocity at a point decreases/increases with increase in the magnetic field when the free stream velocity is less/greater than the stretching velocity. It is further observed that for a given value of magnetic parameter M, the dimensionless shear stress coefficient $|F''(0)|$ increases with increase in power-law index n when the value of the ratio of free stream velocity and stretching velocity is close to 1 but not equal to 1. But when the value of this ratio further differs from 1, the variation of $|F''(0)|$ with n is non-monotonic.

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1. Introduction

Flow of an incompressible viscous fluid over a stretching surface has an important bearing on several technological processes. In particular in the extrusion of a polymer in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a sheet which is then solidified through quenching or gradual cooling by direct contact with water. Further, the study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid caused by the deformation of the walls of the vessel containing this fluid is of considerable interest in modern metallurgical and metal-working processes. Crane [1] gave an exact similarity solution in closed analytical form for steady boundary layer flow of an incompressible viscous fluid caused solely by the stretching of an elastic flat sheet which moves in its own plane with a velocity varying linearly with distance from a fixed point. Pavlov [2] gave an exact similarity solution to the MHD boundary layer equations for the steady two-dimensional flow of an electrically conducting incompressible fluid due to the stretching of a plane elastic surface in the presence of a uniform transverse magnetic field. Andersson [3] investigated the MHD flow of a viscoelastic fluid past a stretching surface in presence of a uniform transverse magnetic field. Andersson and

Dandapat [4] extended the Newtonian boundary layer flow considered by Crane [1] to an important class of non-Newtonian fluids obeying power-law model. MHD flow of a power-law fluid over a stretching surface was examined by Andersson et al. [5]. It is known that several electrically conducting fluids acquire non-Newtonian properties in the presence of strong electric and magnetic fields (see [6]). A comprehensive discussion on the potential engineering applications of non-Newtonian power-law electrically conducting fluids permeated by magnetic fields was presented by Martinson and Pavlov [7].

Recently Chiam [8] studied two-dimensional steady stagnation-point flow of an incompressible viscous fluid towards a stretching surface in the case when the parameter b representing the ratio of the strain rate of the stagnation flow to that of the stretching sheet is equal to unity. Mahapatra and Gupta [9] studied two-dimensional orthogonal stagnation-point flow of an incompressible viscous fluid towards a stretching surface in the general case $b \neq 1$. They found that the structure of the boundary layer depends critically on the value of b. The corresponding problem of two-dimensional stagnation-point flow of a power-law fluid towards a rigid surface was investigated by Kapur and Srivastava [10]. The extension of the same problem to the axisymmetric case was studied by Maiti [11] and later on by Koneru and Manohar [12]. Sapunkov [13] investigated the two-dimensional orthogonal stagnation-point flow of an incompressible electrically conducting power-law fluid towards a rigid surface in the presence of a uniform transverse magnetic field.

Djukic [14] studied the hydromagnetic Hiemenz flow of a power-law fluid towards a rigid plate. Mahapatra and Gupta [15] analyzed the steady two-dimensional orthogonal stagnation-point flow of an incompressible viscous electrically conducting fluid towards a stretching surface, the flow being permeated by a uniform transverse magnetic field.

In this paper we investigate steady, two-dimensional orthogonal stagnation-point flow of an electrically conducting power-law fluid towards a stretching surface in the presence of a uniform transverse magnetic field. The motivation for studying this problem stems from the fact that it may arise in metal-working processes.

2. Flow analysis

Consider the steady two-dimensional stagnation-point flow of an electrically conducting power-law fluid in the presence of a uniform transverse magnetic field towards a flat surface coinciding with the plane $y = 0$, the flow being confined to the region $y > 0$. Two equal and opposing forces are applied on the stretching surface along the x-axis so that the surface is stretched keeping the origin fixed as shown in Fig. 1.

The MHD equations for steady two-dimensional stagnation-point flow in the boundary layer towards the stretching surface are, in the usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y} - \frac{\sigma B_0^2}{\rho}(u - U), \tag{2}$$

where the induced magnetic field is neglected (which is justified for MHD flow at small magnetic Reynolds number [16]). It is also assumed that the external electric field is zero and the electric field due to polarisation of charges is negligible. Here u and v are the velocity components along the x and y direction, respectively. Further ρ, σ, B_0 and τ_{xy} are the density, electrical conductivity, magnetic field and shear stress respectively. In (2), $U(x)$ stands for the stagnation-point velocity in the inviscid free stream. The stress tensor is defined as [17]

$$\tau_{ij} = 2K|2D_{ml}D_{ml}|^{(n-1)/2}D_{ij}, \tag{3}$$

Fig. 1. A sketch of the physical problem.

where

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

denotes the stretching tensor, K is called consistency coefficient and n is the power-law index. Fluids obeying constitutive equation (3) are called power-law fluid. If $n < 1$, the fluid is called pseudoplastic power-law fluid and if $n > 1$, it is called dilatant power-law fluid since the apparent viscosity decreases (shear-thinning) or increases with the increase in shear rate (shear-thickening) accordingly as $n < 1$ or $n > 1$.

In the present problem we have $\partial u/\partial y < 0$ when $a/c < 1$ and $\partial u/\partial y > 0$ when $a/c > 1$. This gives shear stress as

$$\tau_{xy} = -K\left(-\frac{\partial u}{\partial y}\right)^n \quad \text{when } a/c < 1 \tag{5}$$

and

$$\tau_{xy} = K\left(\frac{\partial u}{\partial y}\right)^n \quad \text{when } a/c > 1. \tag{6}$$

Now the momentum equation (2) becomes, when $a/c < 1$,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} - \frac{K}{\rho}\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n - \frac{\sigma B_0^2}{\rho}(u - U) \tag{7}$$

and when $a/c > 1$,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma B_0^2}{\rho}(u - U). \tag{8}$$

The appropriate boundary conditions are

$$u = cx, \quad v = 0 \quad \text{at } y = 0, \tag{9}$$

$$u \to U(x) = ax, \quad v = -ay \text{ as } \quad y \to \infty, \tag{10}$$

where a and c are positive constants. Under the transformations

$$\psi = \left(\frac{K/\rho}{c^{1-2n}}\right)^{1/(n+1)} x^{2n/(n+1)} F(\eta) \tag{11}$$

and

$$\eta = y\left(\frac{c^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{(1-n)/(1+n)}, \tag{12}$$

where

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{13}$$

define the stream function ψ, the governing equations (7) and (8) become, when $a/c < 1$,

$$n[-F''(\eta)]^{(n-1)}F'''(\eta) + \left(\frac{2n}{n+1}\right)F(\eta)F''(\eta)$$
$$- F'^2(\eta) - MF'(\eta) + M\frac{a}{c} + \frac{a^2}{c^2} = 0 \tag{14}$$

and when $a/c > 1$,

$$n[F''(\eta)]^{(n-1)}F'''(\eta) + \left(\frac{2n}{n+1}\right)F(\eta)F''(\eta)$$
$$- F'^2(\eta) - MF'(\eta) + M\frac{a}{c} + \frac{a^2}{c^2} = 0. \tag{15}$$

The boundary conditions are

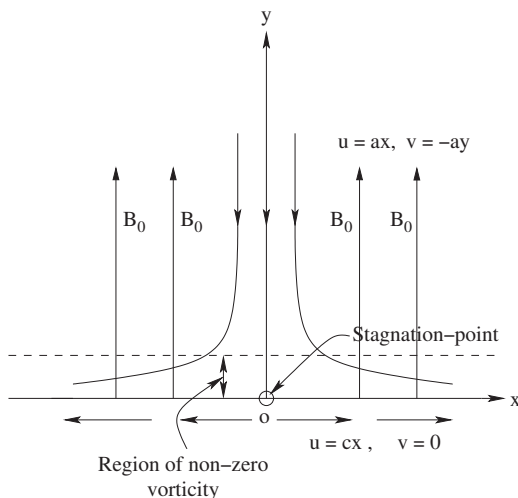$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = a/c, \tag{16}$$

where the prime denotes differentiation with respect to η and $M = \sigma B_0^2/(\rho c)$ is the magnetic parameter. The skin friction coefficient C_f at the wall is given by

$$C_f = \frac{\tau_w}{(1/2)\rho(cx)^2} = 2[-F''(0)]^n \left[\frac{(cx)^{2-n}x^n}{K/\rho}\right]^{-1/(1+n)}$$

$$\text{when } a/c < 1 \tag{17}$$

and

$$C_f = 2[F''(0)]^n \left[\frac{(cx)^{2-n}x^n}{K/\rho}\right]^{-1/(1+n)} \quad \text{when } a/c > 1, \tag{18}$$

where $(cx)^{2-n}x^n/(K/\rho)$ is the local Reynolds number based on the sheet velocity cx.

3. Uniqueness

We now study the uniqueness of the solution to the boundary value problem (BVP) (14)–(16). It may be noted in this connection that the uniqueness of the steady flow of an incompressible viscous fluid past a stretching sheet in the presence of a uniform free stream was established by McLeod and Rajagopal [18]. Further the existence and uniqueness of the solution to steady two-dimensional orthogonal stagnation-point flow of an incompressible electrically non-conducting fluid in the absence of any magnetic field was investigated by Paullet and Weidman [19].

We begin by considering the governing equation for MHD stagnation-point flow of a Newtonian fluid ($n = 1$) which is obtained from (14) or (15) as

$$F'''(\eta) + F(\eta)F''(\eta) - F'^2(\eta) - MF'(\eta) + Mb + b^2 = 0, \tag{19}$$

subject to boundary conditions

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = b, \tag{20}$$

where $b = a/c$.

3.1. The case $b > 1$

We first show that if $b > 1$, then any solution to the BVP under consideration must necessarily be monotonic. We take

$$F''(0) = \alpha, \tag{21}$$

where α is a free parameter and put

$$v(\eta, \alpha) = \frac{\partial F}{\partial \alpha}. \tag{22}$$

Differentiating (19) with respect to α, we get

$$v''' + Fv'' + vF'' - 2F'v' - Mv' = 0, \tag{23}$$

subject to

$$v(0) = 0, \quad v'(0) = 0, \quad v''(0) = 1, \quad v'''(0) = 0. \tag{24}$$

Again differentiating (23) with respect to η, we get

$$v^{(iv)} + Fv''' - (F' + M)v'' - F''v' + F'''v = 0. \tag{25}$$

Hence using (20) and (24), we have

$$v^{(iv)}(0) = 1 + M > 0. \tag{26}$$

Now F' can be a maximum when $F'' = 0$ and $F''' < 0$ at $\eta = \eta_{max}$. Then from (19) at the maximum point of F', we have

$F''' = F'^2 + MF' - Mb - b^2 < 0.$

This implies $(F' - b)(F' + b + M) < 0.$

Hence either (i) $F' - b > 0$, $F' + b + M < 0$ or (ii) $F' - b < 0$, $F' + b + M > 0$.

Now if $F' > b$, then clearly $F' + b + M > 0$, which contradicts $F' + b + M < 0$. Thus (i) is ruled out. Hence (ii) holds so that $-(b + M) < F' < b$. Thus any solution that rises above b cannot be a solution to (19).

Next consider the case that F' has a maximum below b. At such point $F' < b$, $F'' = 0$ and $F''' < 0$. After the maximum below b, to satisfy the boundary condition at ∞, F' must turn concave up ($F''' > 0$) and have a minimum and then at a later point turn concave down ($F''' < 0$) as $F' \to b$ from below. Since F''' goes from negative to positive and back to negative, F''' at some point must have a positive maximum. At this point $F''' > 0$, $F^{(iv)} = 0$ and $F^{(v)} < 0$. Differentiating (19), we get

$$F^{(iv)} + FF''' - F'F'' - MF'' = 0. \tag{27}$$

Differentiating (27) again, we have

$$F^{(v)} + FF^{(iv)} - F''^2 - MF''' = 0. \tag{28}$$

Hence at the above maximum point,

$$F^{(v)} - F''^2 - MF''' = 0. \tag{29}$$

This gives $F^{(v)} = F''^2 + MF''' > 0$. This is a contradiction since we have seen that $F^{(v)} < 0$. Thus no solution to the BVP has a maximum below $F' = b$.

We next eliminate the possibility of a minimum of F'. In fact at a minimum of F', we have from (19), $F''' = F'^2 + MF' - Mb - b^2 > 0$.

This implies $(F' - b)(F' + b + M) > 0.$

Hence either (i) $F' - b > 0$, $F' + b + M > 0$ or (ii) $F' - b < 0$, $F' + b + M < 0$

Now minimum of F' cannot occur above $F' = b$, since $F'(0) = 1 < b$ (we assume $b > 1$). In order that F' has a minimum above $F' = b$, there ought to be a maximum of F' above $F' = b$. But we have already seen that there cannot be any maximum of F' above $F' = b$. Hence any minimum of F' must occur with $F' < -(b + M)$.

Again $F'''(0) = (1 - b)(1 + b + M) < 0$ since $b > 1$. Thus F' starts off concave down, must turn concave up at its minimum and then concave down again to fulfil the boundary condition at ∞. We have already seen from above that this is not possible. Hence if $b > 1$, any solution to the BVP must be monotonic.

To show uniqueness, suppose that there exist two solutions such that $F'(\eta, \alpha_1)$ and $F'(\eta, \alpha_2)$ are both solutions with $\alpha_2 > \alpha_1$.

Lemma 1. *For all $\eta > 0$ and all $\alpha_1 \leqslant \alpha \leqslant \alpha_2$, we have $v' > 0$. Also v' is bounded away from zero as $\eta \to \infty$.*

Proof. From the initial conditions on v given by (24) and (26), one can show that there exists an interval $0 < \eta < \varepsilon$ such that initially in $0 < \eta < \varepsilon$, $v' > 0$, $v'' > 1$ and $v''' > 0$. Since $v''' > 0$ initially, v' is initially positive and concave up. For v' ever to become zero, v'-curve must change from concave up to concave down. Thus there exists a point η_1 with $v > 0$, $v' > 0$, $v'' > 0$, $v''' = 0$ and $v^{(iv)} \leqslant 0$. Until this point, v and all its derivatives up to v''' are positive. Hence F and all its derivatives through F''' are increasing function of α. Hence in the interval $\alpha_1 \leqslant \alpha \leqslant \alpha_2$, we must have

$$0 < F(\eta; \alpha_1) \leqslant F(\eta; \alpha) \leqslant F(\eta; \alpha_2), \tag{30}$$

$$1 < F'(\eta; \alpha_1) \leqslant F'(\eta; \alpha) \leqslant F'(\eta; \alpha_2) < b, \tag{31}$$

$$0 < F''(\eta; \alpha_1) \leqslant F''(\eta; \alpha) \leqslant F''(\eta; \alpha_2) \tag{32}$$

and

$$F'''(\eta; \alpha_1) \leqslant F'''(\eta; \alpha) \leqslant F'''(\eta; \alpha_2) \tag{33}$$

for any $0 < \eta \leqslant \eta_1$. Now from (25),

$$v^{(iv)}(\eta_1) = (F'(\eta_1) + M) v''(\eta_1) + F''(\eta_1)v'(\eta_1)$$
$$- F'''(\eta_1)v(\eta_1). \qquad (34)$$

It follows from (19) that if $F'''(\eta) = 0$, then

$$FF'' + (b + F')(b - F') + M(b - F') = 0. \qquad (35)$$

Now since $1 < F' < b$, $F > 0$, $F'' > 0$ for $\eta > 0$, we find from (35) that for $\eta > 0$, there is a contradiction since each term in (35) is positive. Hence $F''' < 0$ for $\eta > 0$.

It now follows from (34) that each term on the right-hand side of (34) is positive, so that $v^{(iv)}(\eta_1) > 0$. This contradicts the fact that $v^{(iv)}(\eta_1) \leqslant 0$. Hence v' is positive for all $\eta > 0$ and $\alpha_1 \leqslant \alpha \leqslant \alpha_2$. Further $v''' > 0$ and can never vanish. Since $v'(0) = 0$, $v''(0) = 1$, we infer that v' remains bounded away from zero as $\eta \to \infty$. $\square$

Using mean value theorem, we get

$$F'(\eta; \alpha_2) - F'(\eta; \alpha_1) = \left(\frac{\partial F'}{\partial \alpha}\right)_{\alpha = \bar{\alpha}} (\alpha_2 - \alpha_1)$$
$$= v'(\eta, \bar{\alpha})(\alpha_2 - \alpha_1), \qquad (36)$$

where $\alpha_1 < \bar{\alpha} < \alpha_2$. Thus as $\eta \to \infty$ in (36) we get

$$0 = b - b = F'(\infty; \alpha_2) - F'(\infty; \alpha_1)$$
$$= v'(\infty; \bar{\alpha})(\alpha_2 - \alpha_1) > 0 \quad \text{for } \alpha_1 \neq \alpha_2. \qquad (37)$$

Thus we arrive at a contradiction. Hence for $b > 1$, the solution to the BVP under consideration is unique.

3.2. The case $0 < b < 1$

Theorem 1. *In this case there can be at most one monotonic solution to the BVP* (19) *and* (20).

Proof. As before we consider the function $v(\eta; \alpha) = \partial F/\partial \alpha$, with $F''(0) = \alpha$ which satisfies Eqs. (23) and (24). Now when $0 < b < 1$, we have from (19) that $F > 0$, $b < F' < 1$, $F'' < 0$, $F''' > 0$ for all $\eta > 0$. Note that for $0 < b < 1$, F' is monotonic decreasing and hence $F'' < 0$.

If possible let there be two monotone solutions $F(\eta; \alpha_3)$ and $F(\eta; \alpha_4)$ with $\alpha_4 > \alpha_3$. $\square$

Lemma 2. *For $\eta > 0$, and all $\alpha_3 \leqslant \alpha \leqslant \alpha_4$, one can show that $v' > 0$ and bounded away from zero as $\eta \to \infty$.*

Proof. Since $v'(0) = 0$ and $v''(0) = 1$, for $\eta > 0$ and small, we must have $v' > 0$ and increasing. This follows from the fact that $v'''(0) = 0$ and this gives from (25), $v^{(iv)}(0) = 1 + M > 0$. Thus we have $v''(\eta) > 0$ and so v' is increasing. Now v' cannot be a maximum because at such a point, $v' > 0$, $v'' = 0$ and $v''' \leqslant 0$. Then at this point, Eq. (23) implies $v''' = -vF'' + 2F'v' + Mv' > 0$, since $F'' < 0$, $v > 0$, $F' > 0$ and $v' > 0$. Thus we arrive at a contradiction since $v''' \leqslant 0$. Hence v' is bounded away from zero as $\eta \to \infty$. $\square$

Finally, by mean value theorem,

$$F'(\eta; \alpha_4) - F'(\eta; \alpha_3) = \left(\frac{\partial F'}{\partial \alpha}\right)_{\alpha = \hat{\alpha}} (\alpha_4 - \alpha_3)$$
$$= v'(\eta, \hat{\alpha})(\alpha_4 - \alpha_3), \qquad (38)$$

where $\alpha_3 < \hat{\alpha} < \alpha_4$. Thus as $\eta \to \infty$ in (38) we get

$$0 = b - b = F'(\infty; \alpha_4) - F'(\infty; \alpha_3)$$
$$= v'(\infty; \hat{\alpha})(\alpha_4 - \alpha_3) > 0 \quad \text{for } \alpha_3 \neq \alpha_4. \qquad (39)$$

We arrive at a contradiction. Hence for $0 < b < 1$, there is at most one monotonic solution to the BVP (19) and (20).

It is to be noted that we have discussed the uniqueness of the solution to the BVP (19) and (20) for Newtonian ($n = 1$) case. Similar results hold for other values of the power-law index n which are omitted here.

4. Numerical solution method

The transformed momentum equations (14) and (15) subject to the boundary conditions (16) are solved numerically by an efficient Shooting method for different values of the parameters n and M. First, however, Eqs. (14) and (15) are written as a system of three first-order differential equations, which are solved by means of a standard fourth-order Runge–Kutta integration technique. Then a Newton iteration procedure is employed to assure quadratic convergence of the iterations required to satisfy outer boundary condition $F'(\infty) = a/c$.

5. Results and discussions

The above problem of stagnation-point MHD flow towards a stretching surface is solved numerically for five different values of the magnetic parameter ($M \leqslant 2.0$), for eight values of the power-law index in the range $0.4 \leqslant n \leqslant 2.5$ and for five values of a/c. Figs. 2 and 3 represent the variation of x-component of velocity for pseudoplastic fluid ($n = 0.4$) and dilatant fluid ($n = 2.0$), respectively for several values of M and a/c. The common characteristic of these two figures is
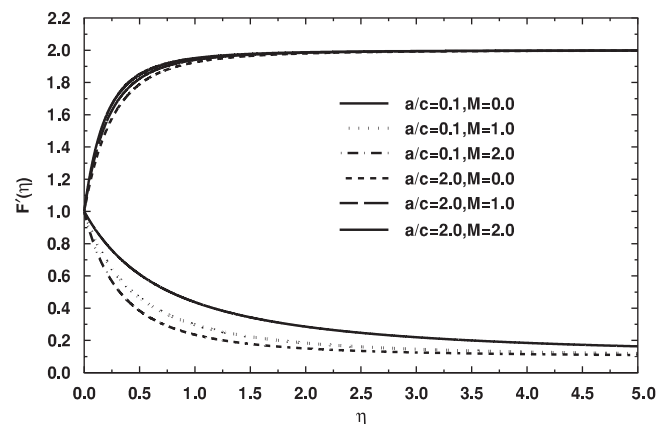
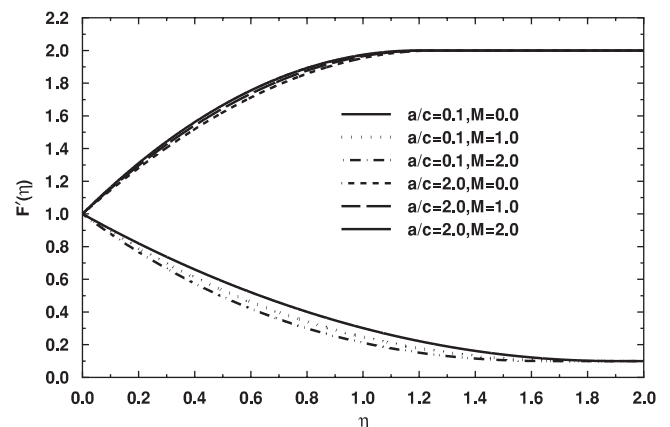**Fig. 2.** Variation of $F'(\eta)$ with η for several values of M and a/c when $n = 0.4$.

Fig. 3. Variation of $F'(\eta)$ with η for several values of M and a/c when $n = 2.0$.

Table 1
Values of $-F''(0)$ for $a/c = 0.2$

n	M				
	0.0	0.5	1.0	1.5	2.0
0.4	1.1593	1.5176	1.8435	2.1474	2.4349
0.6	1.0018	1.2506	1.4725	1.6757	1.8651
0.8	0.9424	1.1356	1.3058	1.4600	1.6021
1.0	0.9181	1.0768	1.2156	1.3404	1.4546
1.2	0.9087	1.0437	1.1612	1.2662	1.3619
1.5	0.9066	1.0170	1.1127	1.1978	1.2749
2.0	0.9133	0.9983	1.0715	1.1363	1.1946
2.5	0.9223	0.9915	1.0510	1.1033	1.1502

Table 2
Values of $-F''(0)$ for $a/c = 0.9$

n	M				
	0.0	0.5	1.0	1.5	2.0
0.4	0.0946	0.1100	0.1247	0.1388	0.1523
0.6	0.1104	0.1253	0.1393	0.1525	0.1650
0.8	0.1314	0.1464	0.1603	0.1734	0.1857
1.0	0.1547	0.1699	0.1839	0.1969	0.2092
1.2	0.1789	0.1943	0.2084	0.2214	0.2336
1.5	0.2155	0.2309	0.2451	0.2581	0.2703
2.0	0.2739	0.2893	0.3033	0.3161	0.3280
2.5	0.3272	0.3422	0.3559	0.3683	0.3798

Table 3
Values of $F''(0)$ for $a/c = 1.1$

n	M				
	0.0	0.5	1.0	1.5	2.0
0.4	0.1035	0.1185	0.1328	0.1465	0.1597
0.6	0.1193	0.1337	0.1471	0.1599	0.1721
0.8	0.1407	0.1549	0.1683	0.1808	0.1927
1.0	0.1643	0.1787	0.1920	0.2045	0.2163
1.2	0.1888	0.2033	0.2167	0.2292	0.2409
1.5	0.2257	0.2402	0.2536	0.2660	0.2777
2.0	0.2844	0.2988	0.3119	0.3241	0.3355
2.5	0.3377	0.3517	0.3645	0.3763	0.3872

Table 4
Values of $F''(0)$ for $a/c = 1.5$

n	M				
	0.0	0.5	1.0	1.5	2.0
0.4	1.2019	1.3432	1.4789	1.6099	1.7368
0.6	1.0174	1.1169	1.2114	1.3019	1.3887
0.8	0.9434	1.0218	1.0958	1.1661	1.2333
1.0	0.9095	0.9750	1.0364	1.0946	1.1499
1.2	0.8937	0.9502	1.0030	1.0529	1.1001
1.5	0.8853	0.9325	0.9765	1.0178	1.0568
2.0	0.8871	0.9244	0.9591	0.9914	1.0218
2.5	0.8946	0.9256	0.9543	0.9810	1.0060

Table 5
Values of $F''(0)$ for $a/c = 2.0$

n	M				
	0.0	0.5	1.0	1.5	2.0
0.4	3.7731	4.1322	4.4796	4.8166	5.1447
0.6	2.7631	2.9827	3.1933	3.3960	3.5919
0.8	2.2893	2.4445	2.5925	2.7342	2.8704
1.0	2.0175	2.1363	2.2491	2.3567	2.4597
1.2	1.8421	1.9377	2.0282	2.1142	2.1963
1.5	1.6714	1.7447	1.8139	1.8794	1.9418
2.0	1.5048	1.5571	1.6062	1.6526	1.6965
2.5	1.4062	1.4465	1.4843	1.5199	1.5536

from the above tables that for a fixed value of M, $|F''(0)|$ increases with increase in n in a small neighborhood (N, say) of $a/c = 1$ excluding $a/c = 1$. But it is interesting to note that $|F''(0)|$ is non-monotonic in a region (P, say) with increase in n outside the region N up to certain values of a/c. However for values of $a/c > 1$ outside the region P, $|F''(0)|$ decreases monotonically with increase in n. For values of $a/c < 1$, the variation of $|F''(0)|$ with n outside the region N is non-monotonic. From the five tables it is evident that for a fixed value of n and a/c, the value of $|F''(0)|$ increases with increase in M. This non-monotonic behavior may perhaps be attributed to the change in the character of the flow as a/c changes its values from less than 1 to greater than 1.

It is interesting to note that when the velocity of the stretching surface is equal to the velocity of the inviscid stream ($a = c$), Eqs. (14) and (15) subject to the boundary condition (16) admit of the exact analytical solution

$$F(\eta) = \eta. \tag{40}$$

From (13) this leads to

$$u = cx = ax \quad \text{and} \quad v = -cy = -ay, \tag{41}$$

which correspond to the velocity distribution in the free stream given by (10). From this we can infer that when $a = c$, the velocity distribution near the stretching surface is the same as that of a flow away from the surface so that no boundary layer is formed near the surface. It should be mentioned here that when $a = c$, the flow is not frictionless in a strict sense. In fact in this case the friction is uniformly distributed and does not, therefore, affect the motion.

Acknowledgments

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that the velocity parallel to the stretching surface decreases with increasing M when $a/c < 1$ and increases with increasing M when $a/c > 1$ for both pseudoplastic and dilatant fluids. Physically this is a consequence of the fact that for a given power-law fluid (pseudoplastic or dilatant), the Lorentz force given by the last term in (2) arising out of the magnetic field has a retarding influence on the flow.

Since $F''(0)$ is negative when $a/c < 1$, the computed variation of $-F''(0)$ with M and n is summarized in Tables 1 and 2 for $a/c = 0.2$ and 0.9, respectively. Again since $F''(0)$ is positive when $a/c > 1$, the computed variation of $F''(0)$ with M and n is summarized in Tables 3, 4 and 5 for $a/c = 1.1$, 1.5 and 2.0, respectively. It can be seen

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