In[5]:= **DSolve[{D[f[x, y, z], x] + D[f[x, y, z], y] + D[f[x, y, z], z] == f[x, y, z],**
f[a + b, a - b, 1] == a * b}, f, {x, y, z}]

Out[5]= DSolve$\left[\left\{f^{(0,0,1)}[x, y, z] + f^{(0,1,0)}[x, y, z] + f^{(1,0,0)}[x, y, z] == f[x, y, z],\right.\right.$
$\left.\left. f[a + b, a - b, 1] == a\, b\right\}, f, \{x, y, z\}\right]$

In[6]:= **DSolve$\left[\left\{f^{(0,0,1)}[x, y, z] + f^{(0,1,0)}[x, y, z] + f^{(1,0,0)}[x, y, z] == f[x, y, z],\right.\right.$**
$\left.\left. f[2 * a, 0, 1] == 2 * a\right\}, f, \{x, y, z\}\right]$

Out[6]= DSolve$\left[\left\{f^{(0,0,1)}[x, y, z] + f^{(0,1,0)}[x, y, z] + f^{(1,0,0)}[x, y, z] == f[x, y, z],\right.\right.$
$\left.\left. f[2 a, 0, 1] == 2 a\right\}, f, \{x, y, z\}\right]$

In[7]:= **DSolve[{3 * (u[x, y] - y) ^ 2 * D[u[x, y], x] - D[u[x, y], y] == 0, u[0, a] == a}, u, {x, y}]**

Out[7]= DSolve$\left[\left\{-u^{(0,1)}[x, y] + 3\,(-y + u[x, y])^2\, u^{(1,0)}[x, y] == 0, u[0, a] == a\right\}, u, \{x, y\}\right]$

In[10]:= **DSolve[{D[u[x, t], t, t] - c^2 * D[u[x, t], x, x] == 0, u[0, t] == 0, u[l, t] == 0},**
u, {x, t}, Assumptions → 0 <= x <= l]

Out[10]= DSolve$\left[\left\{u^{(0,2)}[x, t] - c^2\, u^{(2,0)}[x, t] == 0, u[0, t] == 0, u[l, t] == 0\right\},\right.$
$\left. u, \{x, t\}, \text{Assumptions} \rightarrow 0 \leq x \leq l\right]$

In[15]:= **DSolve[{D[u[x, t], t] - k * D[u[x, t], x, x] == 0, u[l, t] == 0, u[x, 0] == f[x],**
Derivative[1, 0][u][0, t] == 0}, u, {x, t}, Assumptions → 0 ≤ x ≤ l]

Out[15]= $\left\{\left\{u \rightarrow \text{Function}\left[\{x, t\},\right.\right.\right.$
$\frac{1}{l}\,2\,\sum_{K[1]=0}^{\infty}\,e^{-\frac{k\pi^2 t\,(1+2\,K[1])^2}{4\,l^2}}\,\text{Cos}\left[\frac{\pi\,x\,(1 + 2\,K[1])}{2\,l}\right]\,\text{Integrate}\left[\text{Cos}\left[\frac{\pi\,(1 + 2\,K[1])\,K[2]}{2\,l}\right]\,f[K[2]],\right.$
$\left.\left.\left.\left. \{K[2], 0, l\}, \text{Assumptions} \rightarrow k > 0\ \&\&\ l > 0\ \&\&\ t > 0\ \&\&\ 0 \leq x \leq l\right]\right]\right\}\right\}$

In[21]:= **DSolve$\bigl[$**
$\left\{\text{D}[u[r, \theta], r, r] + 1/r * \text{D}[u[r, \theta], r] + 1\big/r^2 * \text{D}[u[r, \theta], \theta, \theta] == 0, u[R, \theta] == f[\theta],\right.$
$\left. u[r, 0] == u[r, 2 * \pi], \text{Derivative}[0, 1][u][r, 0] == \text{Derivative}[0, 1][u][r, 2 * \pi]\right\},$
u, {r, θ}, Assumptions → 0 ≤ r ≤ R && 0 ≤ θ ≤ 2 * $\pi\bigr]$

Out[21]= DSolve$\left[\left\{\frac{u^{(0,2)}[r, \theta]}{r^2} + \frac{u^{(1,0)}[r, \theta]}{r} + u^{(2,0)}[r, \theta] == 0, u[R, \theta] == f[\theta], u[r, 0] == u[r, 2\,\pi],\right.\right.$
$\left.\left. u^{(0,1)}[r, 0] == u^{(0,1)}[r, 2\,\pi]\right\}, u, \{r, \theta\}, \text{Assumptions} \rightarrow 0 \leq r \leq R\ \&\&\ 0 \leq \theta \leq 2\,\pi\right]$

In[24]:= **DSolve[{D[u[x, t], t] - k * D[u[x, t], x, x] - f[x, t] == 0, u[0, t] == 0,**
u[π, t] == 0, u[x, 0] == 0}, u, {x, t}, Assumptions → 0 ≤ x ≤ π && k > 0]

Out[24]= DSolve$\left[\left\{-f[x, t] + u^{(0,1)}[x, t] - k\, u^{(2,0)}[x, t] == 0, u[0, t] == 0, u[\pi, t] == 0, u[x, 0] == 0\right\},\right.$
$\left. u, \{x, t\}, \text{Assumptions} \rightarrow 0 \leq x \leq \pi\ \&\&\ k > 0\right]$

In[25]:= **DSolve[{D[u[x, t], t, t] - 4 * D[u[x, t], x, x] == 0,**
u[x, 0] == Exp[-x^2], Derivative[0, 1][u][x, 0] == 0}, u, {x, t}]

Out[25]= $\left\{\left\{u \rightarrow \text{Function}\left[\{x, t\}, \frac{1}{2}\,\left(e^{-(-2\,t+x)^2} + e^{-(2\,t+x)^2}\right)\right]\right\}\right\}$

In[26]:= **DSolve[{D[u[x, t], t, t] - 4 * D[u[x, t], x, x] == f[a],**
u[x, 0] == 0, Derivative[0, 1][u][x, 0] == x^2}, u, {x, t}]

Out[26]= $\left\{\left\{u \to \text{Function}\left[\{x, t\}, \frac{1}{4}\left(\frac{16\,t^3}{3} + 4\,t\,x^2 + 2\,t^2\,f[a]\right)\right]\right\}\right\}$

In[28]:= **DSolve$\left[\left\{y * \left(D[m[x, y], x]\^2 - D[m[x, y], y]\^2\right) + m[x, y] * D[m[x, y], y] == 0\right\}, m, \{x, y\}\right]$**

DSolve::nlpde : Solution requested to nonlinear partial differential equation Trying to build a special solution ≫

Out[28]= $\Big\{\Big\{m \to \text{Function}\Big[\{x, y\},$

$-\sqrt{\Big(-\frac{4\,x^3}{27\,C[1]} + \frac{4\,x\,y^2}{3\,C[1]} + \frac{4\,x^2\,C[2]}{9\,C[1]} - \frac{4\,y^2\,C[2]}{3\,C[1]} - \frac{4\,x\,C[2]^2}{9\,C[1]} + \frac{4\,C[2]^3}{27\,C[1]} - \frac{1}{54\,C[1]^2}}$

$\left(\sqrt{\Big(\left(8\,x^3\,C[1] - 72\,x\,y^2\,C[1] - 24\,x^2\,C[1]\,C[2] + 72\,y^2\,C[1]\,C[2] + 24\,x\,C[1]\,C[2]^2 - \right.}\right.$

$8\,C[1]\,C[2]^3\big)^2 - 108\,C[1]^2\,\big(-16\,x^4\,y^2 + 32\,x^2\,y^4 - 16\,y^6 + 64\,x^3\,y^2\,C[2] - 64$

$\left.\left.\left.x\,y^4\,C[2] - 96\,x^2\,y^2\,C[2]^2 + 32\,y^4\,C[2]^2 + 64\,x\,y^2\,C[2]^3 - 16\,y^2\,C[2]^4\big)\right)\right)}\Big]\Big\},$

$\Big\{m \to \text{Function}\Big[\{x, y\}, \sqrt{\Big(-\frac{4\,x^3}{27\,C[1]} + \frac{4\,x\,y^2}{3\,C[1]} + \frac{4\,x^2\,C[2]}{9\,C[1]} - \frac{4\,y^2\,C[2]}{3\,C[1]} -}$

$\frac{4\,x\,C[2]^2}{9\,C[1]} + \frac{4\,C[2]^3}{27\,C[1]} - \frac{1}{54\,C[1]^2}$

$\left(\sqrt{\Big(\left(8\,x^3\,C[1] - 72\,x\,y^2\,C[1] - 24\,x^2\,C[1]\,C[2] + 72\,y^2\,C[1]\,C[2] + 24\,x\,C[1]\,C[2]^2 - 8\right.}\right.$

$C[1]\,C[2]^3\big)^2 - 108\,C[1]^2\,\big(-16\,x^4\,y^2 + 32\,x^2\,y^4 - 16\,y^6 + 64\,x^3\,y^2\,C[2] -$

$\left.\left.\left.64\,x\,y^4\,C[2] - 96\,x^2\,y^2\,C[2]^2 + 32\,y^4\,C[2]^2 + 64\,x\,y^2\,C[2]^3 - 16\,y^2\,C[2]^4\big)\right)\right)}\Big]\Big\},$

$\Big\{m \to \text{Function}\Big[\{x, y\}, -\sqrt{\Big(-\frac{4\,x^3}{27\,C[1]} + \frac{4\,x\,y^2}{3\,C[1]} + \frac{4\,x^2\,C[2]}{9\,C[1]} - \frac{4\,y^2\,C[2]}{3\,C[1]} -}$

$\frac{4\,x\,C[2]^2}{9\,C[1]} + \frac{4\,C[2]^3}{27\,C[1]} + \frac{1}{54\,C[1]^2}$

$\left(\sqrt{\Big(\left(8\,x^3\,C[1] - 72\,x\,y^2\,C[1] - 24\,x^2\,C[1]\,C[2] + 72\,y^2\,C[1]\,C[2] + 24\,x\,C[1]\,C[2]^2 - \right.}\right.$

$8\,C[1]\,C[2]^3\big)^2 - 108\,C[1]^2\,\big(-16\,x^4\,y^2 + 32\,x^2\,y^4 - 16\,y^6 + 64\,x^3\,y^2\,C[2] - 64$

$\left.\left.\left.x\,y^4\,C[2] - 96\,x^2\,y^2\,C[2]^2 + 32\,y^4\,C[2]^2 + 64\,x\,y^2\,C[2]^3 - 16\,y^2\,C[2]^4\big)\right)\right)}\Big]\Big\},$

$\Big\{m \to \text{Function}\Big[\{x, y\}, \sqrt{\Big(-\frac{4\,x^3}{27\,C[1]} + \frac{4\,x\,y^2}{3\,C[1]} + \frac{4\,x^2\,C[2]}{9\,C[1]} - \frac{4\,y^2\,C[2]}{3\,C[1]} -}$

$\frac{4\,x\,C[2]^2}{9\,C[1]} + \frac{4\,C[2]^3}{27\,C[1]} + \frac{1}{54\,C[1]^2}$

$\left(\sqrt{\Big(\left(8\,x^3\,C[1] - 72\,x\,y^2\,C[1] - 24\,x^2\,C[1]\,C[2] + 72\,y^2\,C[1]\,C[2] + 24\,x\,C[1]\,C[2]^2 - 8\right.}\right.$

$C[1]\,C[2]^3\big)^2 - 108\,C[1]^2\,\big(-16\,x^4\,y^2 + 32\,x^2\,y^4 - 16\,y^6 + 64\,x^3\,y^2\,C[2] -$

$\left.\left.\left.64\,x\,y^4\,C[2] - 96\,x^2\,y^2\,C[2]^2 + 32\,y^4\,C[2]^2 + 64\,x\,y^2\,C[2]^3 - 16\,y^2\,C[2]^4\big)\right)\right)}\Big]\Big\}\Big\}$