```
> restart;
> with(plots) :
>
> Eq1 := diff(f(η), η$3) + diff(f(η), η$2)·f(η) − diff(f(η), η)^2 + 4 = 0;
```

$$Eq1 := \frac{d^3}{d\eta^3} f(\eta) + \left(\frac{d^2}{d\eta^2} f(\eta)\right) f(\eta) - \left(\frac{d}{d\eta} f(\eta)\right)^2 + 4 = 0 \quad (1)$$

```
>
> Eq2 := diff(θ(η), η$2) + Pr·diff(θ(η), η)·f(η) = 0;
```

$$Eq2 := \frac{d^2}{d\eta^2} \theta(\eta) + Pr \left(\frac{d}{d\eta} \theta(\eta)\right) f(\eta) = 0 \quad (2)$$

```
>
> VPr := [0.01, 0.02, 0.03] :
> etainf := 27 :
> bcs := D(f)(0) = 0, f(0) = 0, D(θ)(0) = -1, D(f)(etainf) = 2, θ(etainf) = 0;
```

$$bcs := D(f)(0) = 0, f(0) = 0, D(\theta)(0) = -1, D(f)(27) = 2, \theta(27) = 0 \quad (3)$$

```
> dsys := {Eq1, Eq2, bcs} :
> for i from 1 to 3 do
    Pr := VPr[i];
    dsol[i] := dsolve(dsys, numeric);
    print(Pr);
     print(dsol[i](0));
    end do:
```

$$0.01$$

$$\left[\eta = 0., f(\eta) = 0., \frac{d}{d\eta} f(\eta) = 0., \frac{d^2}{d\eta^2} f(\eta) = 3.48628426509404, \theta(\eta) = 9.30585609645247, \frac{d}{d\eta} \theta(\eta) = -1.00000000000000\right]$$

$$0.02$$

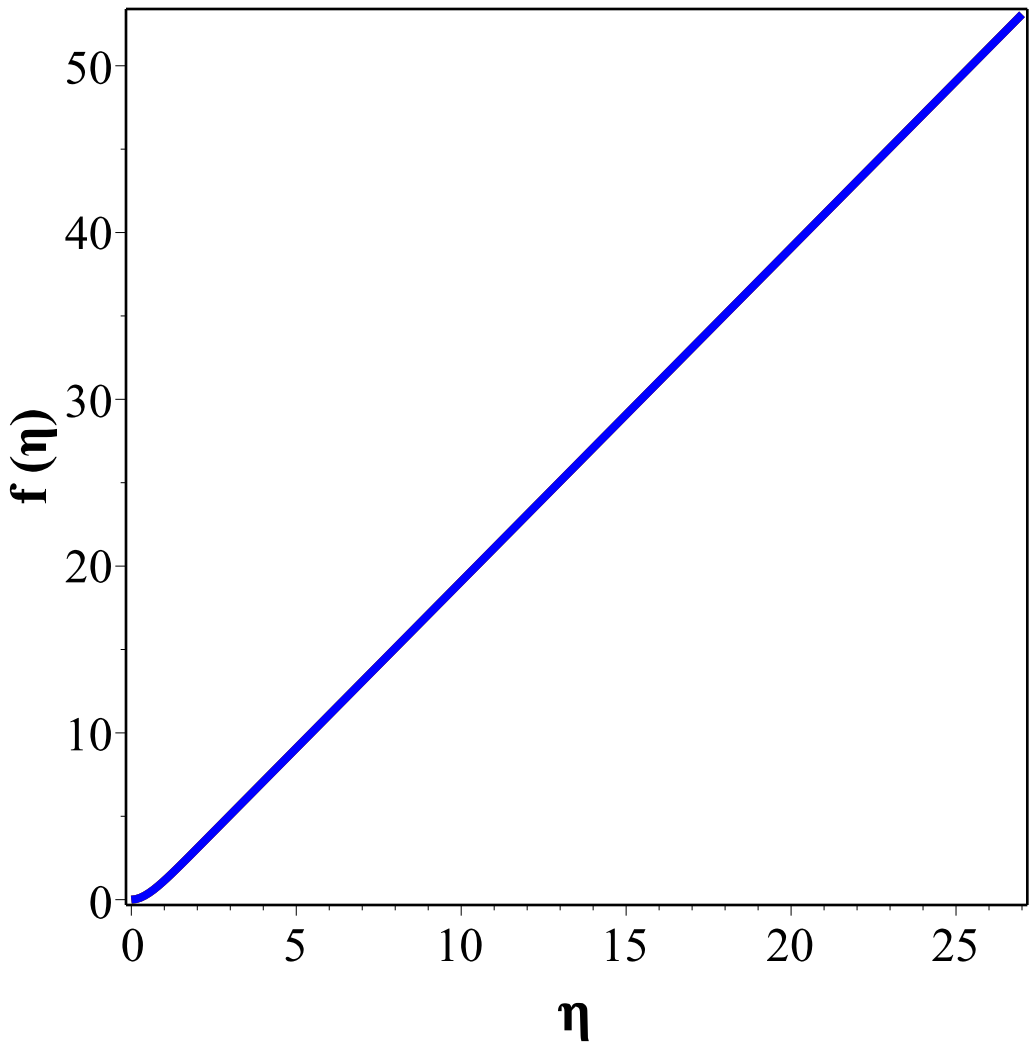
$$\left[\eta = 0., f(\eta) = 0., \frac{d}{d\eta} f(\eta) = 0., \frac{d^2}{d\eta^2} f(\eta) = 3.48628426533922, \theta(\eta) = 6.70646883610737, \frac{d}{d\eta} \theta(\eta) = -1.\right]$$

$$0.03$$

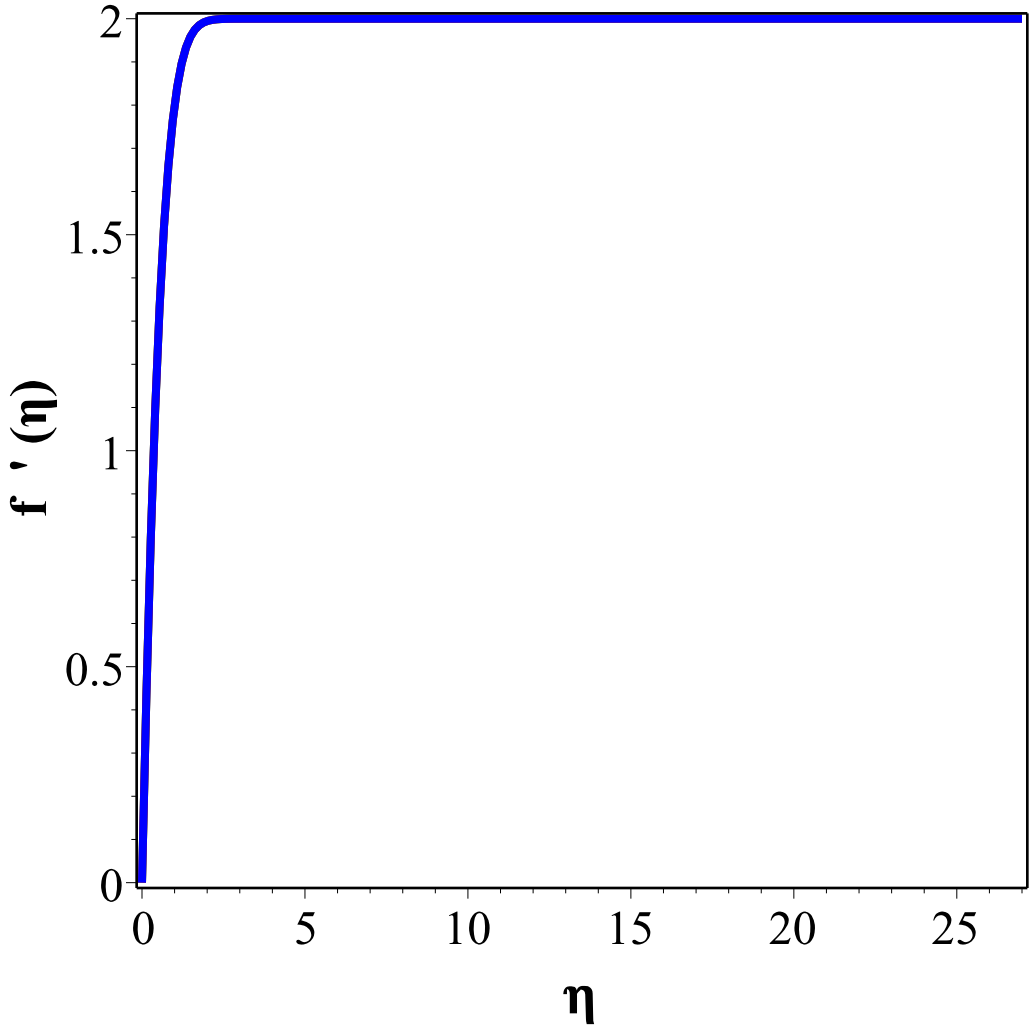
$$\left[\eta = 0., f(\eta) = 0., \frac{d}{d\eta} f(\eta) = 0., \frac{d^2}{d\eta^2} f(\eta) = 3.48628426484592, \theta(\eta) = 5.55258360877069, \frac{d}{d\eta} \theta(\eta) = -1.00000000000000\right] \quad (4)$$

```
> SDf1 := odeplot(dsol[1], [η, f(η)], 0..etainf, color = green, axes = box) :
  SDf2 := odeplot(dsol[2], [η, f(η)], 0..etainf, color = red) :
```

```
SDf3 := odeplot( dsol[3], [η, f(η)], 0..etainf, color = blue) :
> display( [SDf1, SDf2, SDf3 ], labels = ["η", "f (η)"], labeldirections = [horizontal, vertical],
      labelfont = [italic, 16, bold], axes = boxed, axesfont = [times, 14], thickness = 3);
```

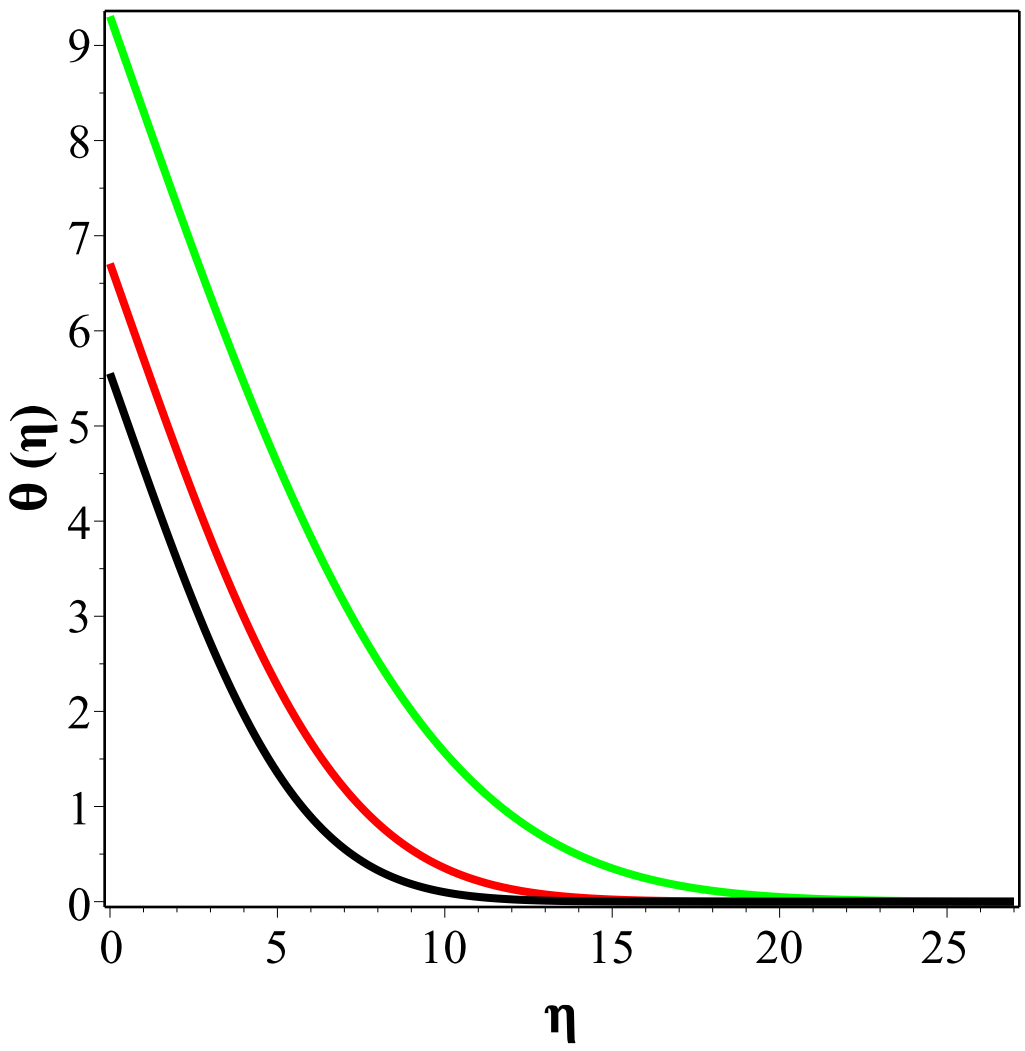



```
>
> SDfd1 := odeplot( dsol[1], [η, diff( f(η), η)], 0..etainf, color = green, axes = box) :
  SDfd2 := odeplot( dsol[2], [η, diff( f(η), η)], 0..etainf, color = red) :
  SDfd3 := odeplot( dsol[3], [η, diff( f(η), η)], 0..etainf, color = blue) :
>
> display( [SDfd1, SDfd2, SDfd3 ], labels = ["η", "f ' (η)"], labeldirections = [horizontal, vertical],
      labelfont = [italic, 16, bold], axes = boxed, axesfont = [times, 14], thickness = 3);
```

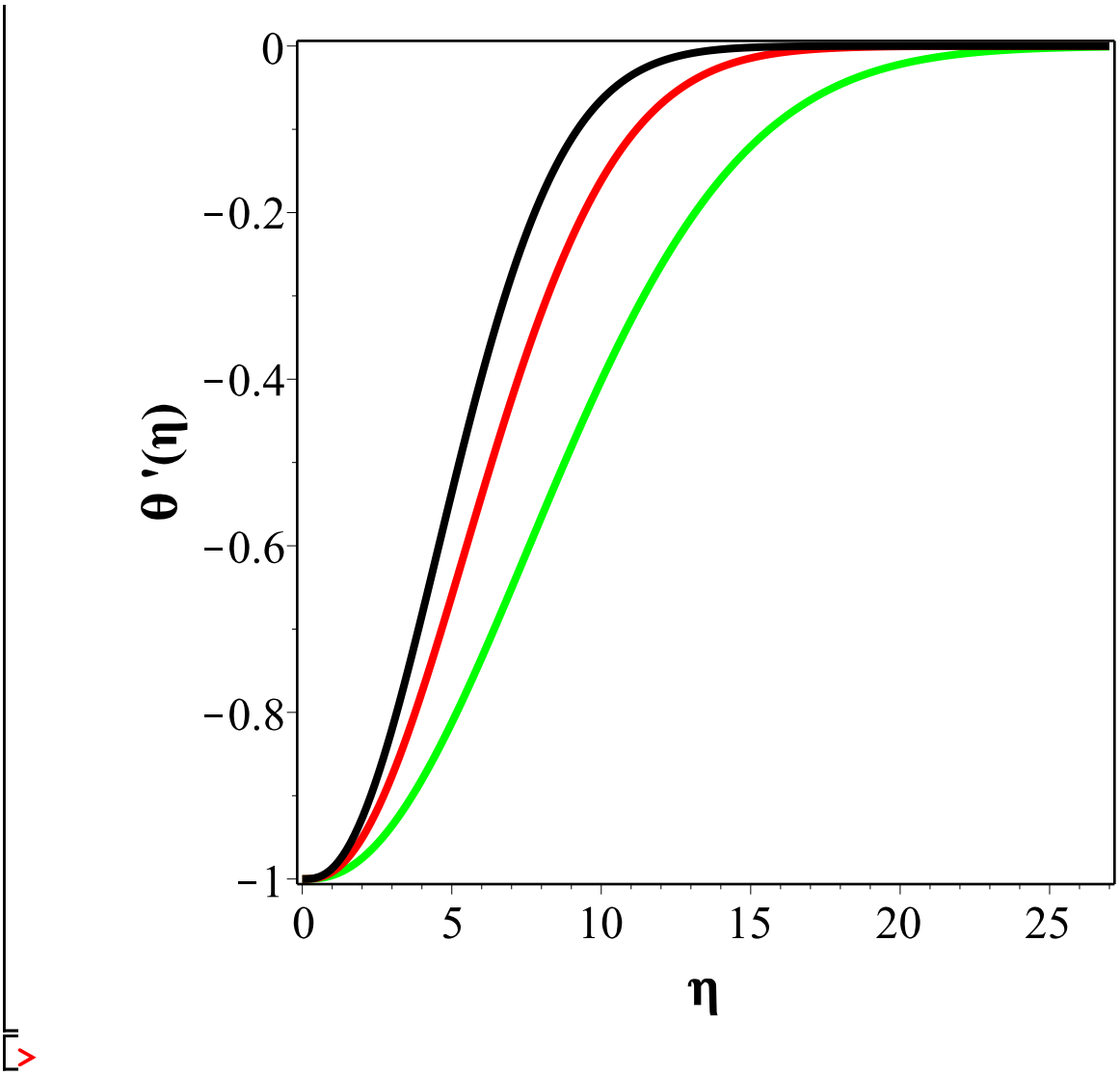



```
> 
> Sθ1 := odeplot(dsol[1], [η, θ(η)], 0..etainf, color = green, axes = box) :
  Sθ2 := odeplot(dsol[2], [η, θ(η)], 0..etainf, color = red) :
  Sθ3 := odeplot(dsol[3], [η, θ(η)], 0..etainf, color = black) :

> display([Sθ1, Sθ2, Sθ3], labels = ["η", "θ (η)"], labeldirections = [horizontal, vertical], labelfont
      = [italic, 16, bold], axes = boxed, axesfont = [times, 14], thickness = 3)
```




```
> 
> Sθd1 := odeplot(dsol[1], [η, diff(θ(η), η)], 0..etainf, color = green, axes = box) :
  Sθd2 := odeplot(dsol[2], [η, diff(θ(η), η)], 0..etainf, color = red) :
  Sθd3 := odeplot(dsol[3], [η, diff(θ(η), η)], 0..etainf, color = black) :
> display([Sθd1, Sθd2, Sθd3], labels = ["η", "θ '(η)"], labeldirections = [horizontal, vertical],
      labelfont = [italic, 16, bold], axes = boxed, axesfont = [times, 14], thickness = 3)
```


0
−0.2
−0.4
−0.6
−0.8
−1
θ '(η)
0
5
10
15
20
25
η