In[9]:= Integrate[Cos[x]^2/(1+Tan[x]),x]

Indefinite integrals:

Hide steps

$$\int \frac{\cos^2(x)}{1+\tan(x)}\,dx = \frac{1}{8}(4x+\sin(2x)+\cos(2x)+2\log(\sin(x)+\cos(x))) + \text{constant}$$

Possible intermediate steps:

Take the integral:

$$\int \frac{\cos^2(x)}{\tan(x)+1}\,dx$$

Multiply numerator and denominator of $\frac{\cos^2(x)}{\tan(x)+1}$ by $\sec^4(x)$:

$$= \int \frac{\sec^2(x)}{\sec^4(x)+\sec^4(x)\tan(x)}\,dx$$

Prepare to substitute $u = \tan(x)$. Rewrite $\frac{\sec^2(x)}{\sec^4(x)+\sec^4(x)\tan(x)}$ using $\sec^2(x) = \tan^2(x)+1$:

$$= \int \frac{\sec^2(x)}{(1+\tan(x))\left(1+\tan^2(x)\right)^2}\,dx$$

For the integrand $\frac{\sec^2(x)}{(1+\tan(x))\left(1+\tan^2(x)\right)^2}$, substitute $u = \tan(x)$ and $du = \sec^2(x)\,dx$:

$$= \int \frac{1}{(u+1)\left(u^2+1\right)^2}\,du$$

For the integrand $\frac{1}{(u+1)\left(u^2+1\right)^2}$, use partial fractions:

$$= \int \left(\frac{1-u}{4\left(u^2+1\right)} + \frac{1-u}{2\left(u^2+1\right)^2} + \frac{1}{4(u+1)}\right)du$$

Integrate the sum term by term and factor out constants:

$$= \frac{1}{4}\int \frac{1-u}{u^2+1}\,du + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Expanding the integrand $\frac{1-u}{u^2+1}$ gives $\frac{1}{u^2+1} - \frac{u}{u^2+1}$:

$$= \frac{1}{4}\int \left(\frac{1}{u^2+1} - \frac{u}{u^2+1}\right)du + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Integrate the sum term by term and factor out constants:

$$= -\frac{1}{4}\int \frac{u}{u^2+1}\,du + \frac{1}{4}\int \frac{1}{u^2+1}\,du + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

For the integrand $\frac{u}{u^2+1}$, substitute $s = u^2+1$ and $ds = 2\,u\,du$:

$$= -\frac{1}{8}\int \frac{1}{s}\,ds + \frac{1}{4}\int \frac{1}{u^2+1}\,du + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

The integral of $\frac{1}{s}$ is $\log(s)$:

$$= -\frac{\log(s)}{8} + \frac{1}{4}\int \frac{1}{u^2+1}\,du + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

The integral of $\frac{1}{u^2+1}$ is $\tan^{-1}(u)$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \frac{1-u}{\left(u^2+1\right)^2}\,du + \frac{1}{4}\int \frac{1}{u+1}\,du$$

For the integrand $\frac{1-u}{\left(u^2+1\right)^2}$, substitute $u = \tan(p)$ and $du = \sec^2(p)\,dp$. Then $\left(u^2+1\right)^2 = \left(\tan^2(p)+1\right)^2$ $= \sec^4(p)$ and $p = \tan^{-1}(u)$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \cos^2(p)\,(1-\tan(p))\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Write $\cos^2(p)$ as $1-\sin^2(p)$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \left(1-\sin^2(p)\right)(1-\tan(p))\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Expanding the integrand $\left(1-\sin^2(p)\right)(1-\tan(p))$ gives $-\sin^2(p) - \tan(p) + \sin^2(p)\tan(p) + 1$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \left(-\sin^2(p) - \tan(p) + \sin^2(p)\tan(p) + 1\right)dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Integrate the sum term by term and factor out constants:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \sin^2(p)\tan(p)\,dp -$$
$$\frac{1}{2}\int \tan(p)\,dp - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Write $\sin^2(p)$ as $1-\cos^2(p)$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int \left(1-\cos^2(p)\right)\tan(p)\,dp -$$

$$\frac{1}{2}\int \tan(p)\,dp - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Expanding the integrand $(1-\cos^2(p))\tan(p)$ gives $\tan(p) - \sin(p)\cos(p)$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int (\tan(p) - \sin(p)\cos(p))\,dp - \frac{1}{2}\int \tan(p)\,dp - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Integrate the sum term by term and factor out constants:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} - \frac{1}{2}\int \sin(p)\cos(p)\,dp - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

For the integrand $\sin(p)\cos(p)$, substitute $w = \cos(p)$ and $dw = -\sin(p)\,dp$:

$$= \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{2}\int w\,dw - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

The integral of w is $\frac{w^2}{2}$:

$$= \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} - \frac{1}{2}\int \sin^2(p)\,dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Write $\sin^2(p)$ as $\frac{1}{2} - \frac{1}{2}\cos(2p)$:

$$= \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} - \frac{1}{2}\int \left(\frac{1}{2} - \frac{1}{2}\cos(2p)\right)dp + \frac{1}{2}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

Integrate the sum term by term and factor out constants:

$$= \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{4}\int \cos(2p)\,dp + \frac{1}{4}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

For the integrand $\cos(2p)$, substitute $v = 2p$ and $dv = 2\,dp$:

$$= \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{1}{8}\int \cos(v)\,dv + \frac{1}{4}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

The integral of $\cos(v)$ is $\sin(v)$:

$$= \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{\sin(v)}{8} + \frac{1}{4}\int 1\,dp + \frac{1}{4}\int \frac{1}{u+1}\,du$$

The integral of 1 is p:

$$= \frac{p}{4} + \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{\sin(v)}{8} + \frac{1}{4}\int \frac{1}{u+1}\,du$$

$$\frac{}{4} \quad \frac{}{4} \quad \frac{}{4} \quad \frac{}{8} \quad \frac{}{8} \quad \frac{}{4}\int \frac{}{u+1}$$

For the integrand $\frac{1}{u+1}$, substitute $z_1 = u + 1$ and $dz_1 = du$:

$$= \frac{p}{4} + \frac{w^2}{4} + \frac{1}{4}\tan^{-1}(u) - \frac{\log(s)}{8} + \frac{\sin(v)}{8} + \frac{1}{4}\int \frac{1}{z_1}\,dz_1$$

The integral of $\frac{1}{z_1}$ is $\log(z_1)$:

$$= \frac{p}{4} - \frac{\log(s)}{8} + \frac{1}{4}\tan^{-1}(u) + \frac{\sin(v)}{8} + \frac{w^2}{4} + \frac{\log(z_1)}{4} + \text{constant}$$

Substitute back for $z_1 = u + 1$:

$$= \frac{p}{4} - \frac{\log(s)}{8} + \frac{1}{4}\log(u+1) + \frac{1}{4}\tan^{-1}(u) + \frac{\sin(v)}{8} + \frac{w^2}{4} + \text{constant}$$

Substitute back for $v = 2\,p$:

$$= \frac{p}{4} + \frac{1}{8}\sin(2\,p) - \frac{\log(s)}{8} + \frac{1}{4}\log(u+1) + \frac{1}{4}\tan^{-1}(u) + \frac{w^2}{4} + \text{constant}$$

Substitute back for $w = \cos(p)$:

$$= \frac{p}{4} + \frac{1}{8}\sin(2\,p) + \frac{\cos^2(p)}{4} - \frac{\log(s)}{8} + \frac{1}{4}\log(u+1) + \frac{1}{4}\tan^{-1}(u) + \text{constant}$$

Substitute back for $p = \tan^{-1}(u)$:

$$= -\frac{\log(s)}{8} + \frac{1}{4}\log(u+1) + \frac{1}{2}\tan^{-1}(u) + \frac{1}{8}\sin\left(2\tan^{-1}(u)\right) + \frac{1}{4}\cos\left(\tan^{-1}(u)\right)^2 + \text{constant}$$

Simplify using $\cos\left(\tan^{-1}(z)\right) = \frac{1}{\sqrt{z^2+1}}$:

$$= \frac{-\left(u^2+1\right)\log(s) + 2\left(\left(u^2+1\right)\log(u+1) + u + 1\right) + 4\left(u^2+1\right)\tan^{-1}(u)}{8\left(u^2+1\right)} + \text{constant}$$

Substitute back for $s = u^2 + 1$:

$$= \frac{1}{8\left(u^2+1\right)}\left(2\left(\left(u^2+1\right)\log(u+1) + u + 1\right) - \left(u^2+1\right)\log\left(u^2+1\right) + 4\left(u^2+1\right)\tan^{-1}(u)\right) + \text{constant}$$

Substitute back for $u = \tan(x)$:

$$= \frac{1}{8}\left(\sin(2\,x) + \cos(2\,x) + 4\tan^{-1}(\tan(x)) + 2\log(\tan(x)+1) - \log\left(\sec^2(x)\right) + 1\right) + \text{constant}$$

Which is equivalent for restricted x values to:

Answer:

$$= \frac{1}{8}(4\,x + \sin(2\,x) + \cos(2\,x) + 2\log(\sin(x) + \cos(x))) + \text{constant}$$

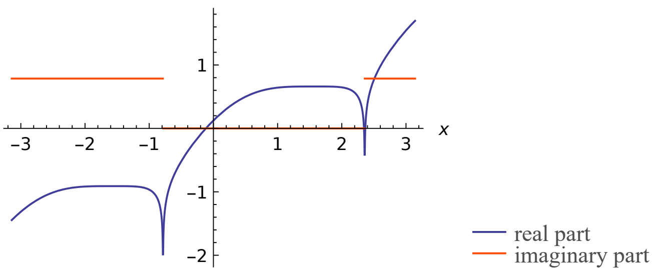
log(x) is the natural logarithm »

sec(x) is the secant function »

$\tan^{-1}(x)$ is the inverse tangent function »

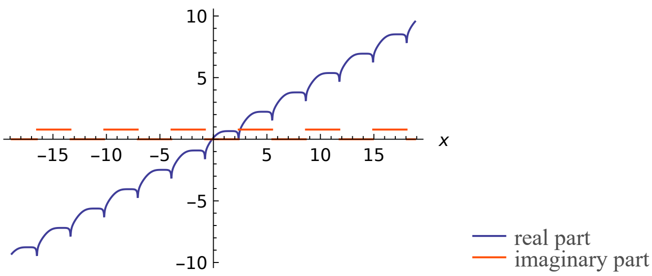
Plots of the integral:

Complex-valued plot



min max

Complex-valued plot



min max

Alternate forms of the integral:

More

$$\frac{1}{8}\sin(2\,x) + \frac{1}{8}(4\,x + \cos(2\,x) + 2\log(\sin(x) + \cos(x))) + \text{constant}$$

$$\frac{1}{8}\left(4\,x + \sqrt{2}\,\sin\left(2\,x + \frac{\pi}{4}\right) + 2\log(\sin(x) + \cos(x))\right) + \text{constant}$$

$$\frac{x}{2} - \frac{\sin^2(x)}{8} + \frac{\cos^2(x)}{8} + \frac{1}{4}\sin(x)\cos(x) + \frac{1}{4}\log(\sin(x) + \cos(x)) + \text{constant}$$

Expanded form of the integral:

Step-by-step solution

$$\frac{x}{2} + \frac{1}{8}\sin(2\,x) + \frac{1}{8}\cos(2\,x) + \frac{1}{4}\log(\sin(x) + \cos(x)) + \text{constant}$$

Series expansion of the integral at x=0:

$$\frac{1}{8} + x - \frac{x^2}{2} - \frac{x^4}{12} + O(x^5)$$

(Taylor series)

Big-O notation »

Series expansion of the integral at x=−π/4:

$$\frac{1}{8}\left(2\log\left(x+\frac{\pi}{4}\right) - \pi - 1 + \log(2)\right) + \frac{3}{4}\left(x+\frac{\pi}{4}\right) + \frac{5}{24}\left(x+\frac{\pi}{4}\right)^2 - \frac{1}{6}\left(x+\frac{\pi}{4}\right)^3 - \frac{61}{720}\left(x+\frac{\pi}{4}\right)^4 + \frac{1}{30}\left(x+\frac{\pi}{4}\right)^5 + O\left(\left(x+\frac{\pi}{4}\right)^6\right)$$

(generalized Puiseux series)

Big-O notation »

Series expansion of the integral at x=(3 π)/4:

$$\frac{1}{8}\left(2\log\left(x-\frac{3\pi}{4}\right) + (3+2i)\pi - 1 + \log(2)\right) + \frac{3}{4}\left(x-\frac{3\pi}{4}\right) + \frac{5}{24}\left(x-\frac{3\pi}{4}\right)^2 - \frac{1}{6}\left(x-\frac{3\pi}{4}\right)^3 - \frac{61}{720}\left(x-\frac{3\pi}{4}\right)^4 + \frac{1}{30}\left(x-\frac{3\pi}{4}\right)^5 + O\left(\left(x-\frac{3\pi}{4}\right)^6\right)$$

(generalized Puiseux series)

Big-O notation »