METHOD TO SOLVE NONLINEAR SCHRODINGER EQUATION BY HOMOTOPY PERTURBATION METHOD.

One dimensional Schrodinger equation with the initial condition.

$$iUt = -\frac{1}{2}U_{xx} - |U|^2$$

Initial condition: $U(X,0) = e^{ix}$

$$U_t - U(X,0) = p(i\left(\frac{1}{2}U_{xx} + |U|^2U\right) - U(\mathrm{X}, 0))$$

$$U_j = i\int_0^t \left(\frac{1}{2}U_{(j-1)xx} + \sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1} |U_i|\,|U_k|\,U_{j-k-i-1}\right)dt$$

$$U_1 = i\int_0^t \left(\frac{1}{2}U_{(0)xx} + \sum_{i=0}^{0}\sum_{k=0}^{0}|U_0|\,|U_0|\,U_0\right)dt$$

$$= i\int_0^t \left(-\frac{1}{2}e^{ix} + |U_0|^2U_0\right)dt$$

$$= i\int_0^t (-\frac{1}{2}e^{ix} + e^{ix})dt$$

$$= i\int_0^t \left(\frac{1}{2}e^{ix}\right)dt$$

$$= i\ \frac{1}{2}e^{ix}$$

$$= \frac{1}{1!}\left(\frac{1}{2}it\right)e^{ix}$$

$$U_2 = i\int_0^t \left(\frac{1}{2}U_{(1)xx} + \sum_{i=0}^{1}\sum_{k=0}^{1}|U_0|\,|U_0|\,U_1\right)dt$$

$$= i\int_0^t \left(\frac{1}{2}(-i\frac{1}{2}\ te^{ix}) + |U_0|^2 i\frac{1}{2}te^{ix}\right)dt$$

$$= i\int_0^t \left(-i\frac{1}{4}\ te^{ix}) + i\frac{1}{2}te^{ix}\right)dt$$

$$= i(\ -i\frac{1}{8}t^2e^{ix} + i\frac{1}{4}t^2e^{ix})$$

$$= i\left(i\ \frac{1}{8}t^2e^{ix}\right)\text{S}$$

$$= -\frac{1}{8}t^2e^{ix}$$

$$= \frac{1}{2!}(\frac{1}{2}it)^2e^{ix}$$

$$U_3 = i\int_0^t \left(\frac{1}{2}U_{(3)xx} + \sum_{i=0}^{2}\sum_{k=0}^{2}|U_0|\,|U_0|\,U_2\right)dt$$

$$= i\int_0^t(\frac{1}{2}\left(\frac{1}{8}t^2e^{ix}\right) + |U_0|^2 - \frac{1}{8}t^2e^{ix})dt$$

$$= i\int_0^t \left(\frac{1}{16}t^2e^{ix} - \frac{1}{8}t^2e^{ix}\right)dt$$

$$= i(\frac{1}{48}t^3e^{ix} - \frac{1}{24}t^3e^{ix})$$

$$= i(-\frac{1}{48}t^3e^{ix})$$

$$= -i\frac{1}{48}t^3e^{ix}$$

$$= \frac{1}{3!}(\frac{1}{2}it)^3e^{ix}$$

Exact solution:

$$U(x,t) = \sum_{n=0}^{\infty} U_n(x,t)$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}(\frac{1}{2}it)^n e^{ix}$$

$$= e^{ix}e^{\frac{1}{2}it}$$

$$= e^{i(x+\frac{1}{2}t)}$$