> $d := 10;$

$$d := 10 \tag{1}$$

> $c := 10;$

$$c := 10 \tag{2}$$

> $L1 := 4;$

$$L1 := 4 \tag{3}$$

> $Ri := \frac{d}{2} + L1;$

$$Ri := 9 \tag{4}$$

> $Rf := c + \frac{d}{2} + L1;$

$$Rf := 19 \tag{5}$$

> $R := (\beta) \rightarrow Ri + \left(\frac{2 \cdot \beta \cdot (Rf - Ri)}{\pi} \right);$

$$R := \beta \mapsto Ri + \frac{2\,\beta\,(Rf - Ri)}{\pi} \tag{6}$$

> $rmax := (\theta) \rightarrow \frac{((\ 2.\ L1.\cos(\theta)\ ) + (\ \mathrm{sqrt}(d^2 - 2 \cdot (L1)^2 + 2 \cdot (L1)^2 \cos(2 \cdot \theta))))}{2};$

$$rmax := \theta \mapsto \frac{(2.\ L1) \cdot \cos(\theta)}{2} + \frac{\sqrt{d^2 - 2\,L1^2 + 2\,L1^2 \cos(2\,\theta)}}{2} \tag{7}$$

> $Rmax := (\theta, \beta) \rightarrow \frac{rmax(\theta) \cdot R(\beta)}{Ri};$

$$Rmax := (\theta, \beta) \mapsto \frac{rmax(\theta)\ R(\beta)}{Ri} \tag{8}$$

> $Vm := (\beta) \rightarrow \frac{1}{\int_0^{2 \cdot \pi} \int_0^{Rmax(\theta, \beta)} \left(1 - \frac{r^2}{(Rmax(\theta, \beta))^2}\right) \cdot r\,dr\,d\theta}$

$$Vm := \beta \mapsto \frac{1}{\int_0^{2\,\pi} \int_0^{Rmax(\theta, \beta)} \left(1 - \frac{r^2}{Rmax(\theta, \beta)^2}\right) r\,dr\,d\theta} \tag{9}$$

> $VB := (r, \theta, \beta) \rightarrow Vm(\beta) \cdot \left(1 - \frac{r^2}{(Rmax(\theta, \beta))^2}\right)$

$$VB := (r, \theta, \beta) \mapsto Vm(\beta) \left(1 - \frac{r^2}{Rmax(\theta, \beta)^2}\right) \tag{10}$$

> $de1 := (r \cdot (Rf - r \cdot \cos(\theta)) \cdot diff(Vr(r, \theta, \beta), r)) + ((Rf - 2 \cdot r \cdot \cos(\theta)) \cdot Vr(r, \theta, \beta)) + (r \cdot diff(VB(r, \theta, \beta), \beta)) = 0;$

$$de1 := r\,(19 - r\cos(\theta))\left(\frac{\partial}{\partial r}\,Vr(r,\theta,\beta)\right) + (19 - 2\,r\cos(\theta))\,Vr(r,\theta,\beta) + r\left(\right. \tag{11}$$

$$-\frac{26.26245080\left(1-\frac{81\,r^2}{\left(4.000000000\cos(\theta)+\sqrt{17+8\cos(2\,\theta)}\right)^2\left(9+\frac{20\,\beta}{\pi}\right)^2}\right)}{(9.+6.366197724\,\beta)^3}$$

$$+(6682.979721\,r^2)\Big/\left((9.+6.366197724\,\beta)^2\left(4.000000000\cos(\theta)+\sqrt{17+8\cos(2\,\theta)}\right)^2\left(9+\frac{20\,\beta}{\pi}\right)^3\pi\right)\Bigg) = 0$$

```
> dsolve( {de1, Vr(Rmax(θ, β), θ, β) = 0}, Vr(r, θ, β) );
Error, (in dsolve) unexpected occurrence of the variables {beta,
theta} in the 1st operand of Vr((1/9)*(4*cos(theta)+(17+8*cos(2*
theta))^(1/2))*(9+20*beta/Pi), theta, beta) in the given initial
conditions
>
```