

restart

Test Printing

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SignedArea := **proc** ($a :: \{Vector, list\}, b :: \{Vector, list\}, c :: \{Vector, list, null\} := null$)

local M, A ;

description "signed area of a line segment or 3 points";

if $c = null$ **then**

$A := 1/2 * a[1] * b[2] - 1/2 * a[2] * b[1];$

else

$A := 1/2 * (b[2] - c[2]) * a[1] + 1/2 * (-b[1] + c[1]) * a[2]$
 $+ 1/2 * c[2] * b[1] - 1/2 * c[1] * b[2];$

end if;

return A ;

end proc;

#with(RonanRoutines)

$y := (x) \quad x^2$

$$y := x \quad x^2 \quad (1)$$

$A1 := [t_1, y(t_1)]$

$$A1 := [t_1, t_1^2] \quad (2)$$

$A2 := [t_2, y(t_2)]$

$$A2 := [t_2, t_2^2] \quad (3)$$

$A3 := [t_3, y(t_3)]$

$$A3 := [t_3, t_3^2] \quad (4)$$

$sA1A2A3 := \text{SignedArea}(A1, A2) + \text{SignedArea}(A2, A3) + \text{SignedArea}(A3, A1)$

$$sA1A2A3 := \frac{1}{2} t_1 t_2^2 - \frac{1}{2} t_1^2 t_2 + \frac{1}{2} t_2 t_3^2 - \frac{1}{2} t_2^2 t_3 + \frac{1}{2} t_3 t_1^2 - \frac{1}{2} t_3^2 t_1 \quad (5)$$

factor

$$- \frac{(t_2 - t_3) (t_1 - t_3) (t_1 - t_2)}{2} \quad (6)$$

$x1 := -2.5; x2 := 3.5;$

$t_1 := -2; t_2 := 1; t_3 := 3;$

$iplt := \text{plot}(y(x), x = x1..x2, \text{colour} = \text{blue}) :$

$trplt := \text{plot}([A1, A2, A3, A1], \text{colour} = \text{green},) :$

$\text{plots} := \text{display}(iplt, trplt);$

$\text{unassign}(t_1, t_2, t_3);$

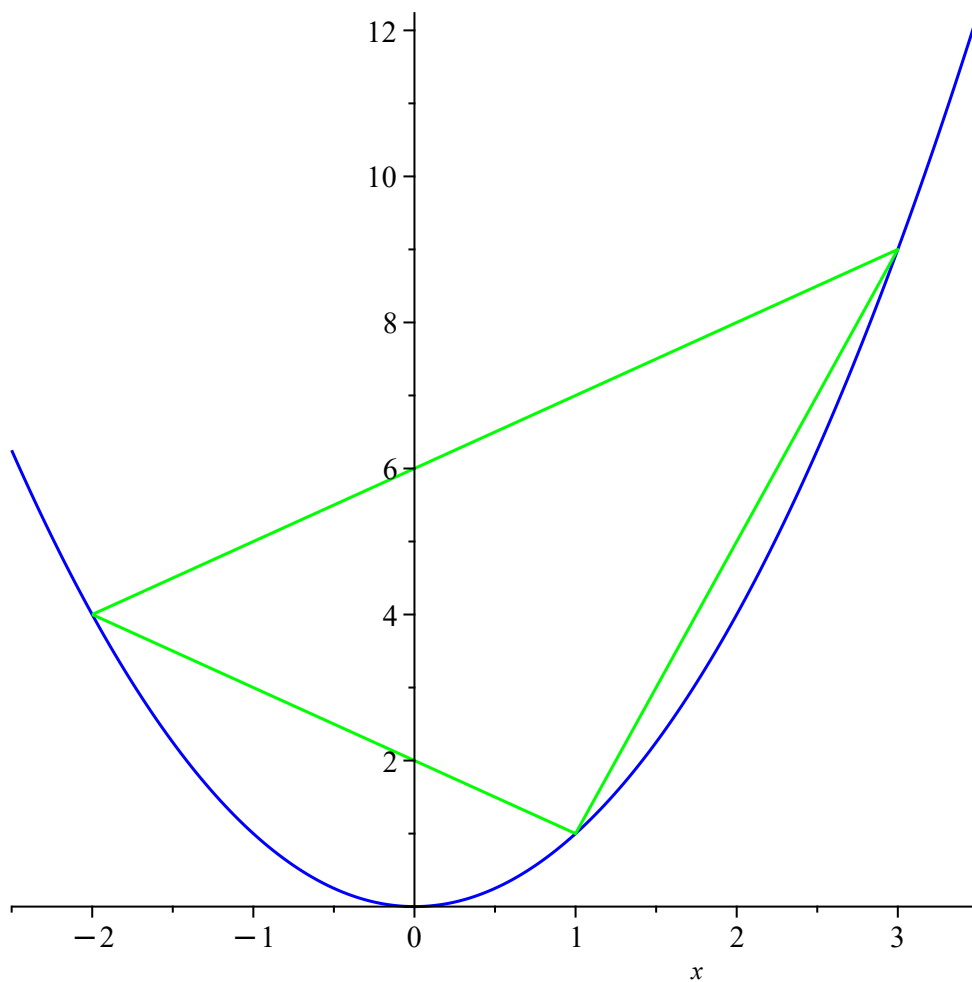
$x1 := -2.5$

$x2 := 3.5$

$t_1 := -2$

$t_2 := 1$

$t_3 := 3$



$$y := (x) \quad x^3 \quad y := x \quad x^3 \quad (7)$$

$$A1 := [t_1, y(t_1)] \quad A1 := [t_1, t_1^3] \quad (8)$$

$$A2 := [t_2, y(t_2)] \quad A2 := [t_2, t_2^3] \quad (9)$$

$$A3 := [t_3, y(t_3)] \quad A3 := [t_3, t_3^3] \quad (10)$$

$$sA1A2A3 := \text{SignedArea}(A1, A2) + \text{SignedArea}(A2, A3) + \text{SignedArea}(A3, A1) \quad sA1A2A3 := \frac{1}{2} t_1 t_2^3 - \frac{1}{2} t_1^3 t_2 + \frac{1}{2} t_2 t_3^3 - \frac{1}{2} t_2^3 t_3 + \frac{1}{2} t_3 t_1^3 - \frac{1}{2} t_3^3 t_1 \quad (11)$$

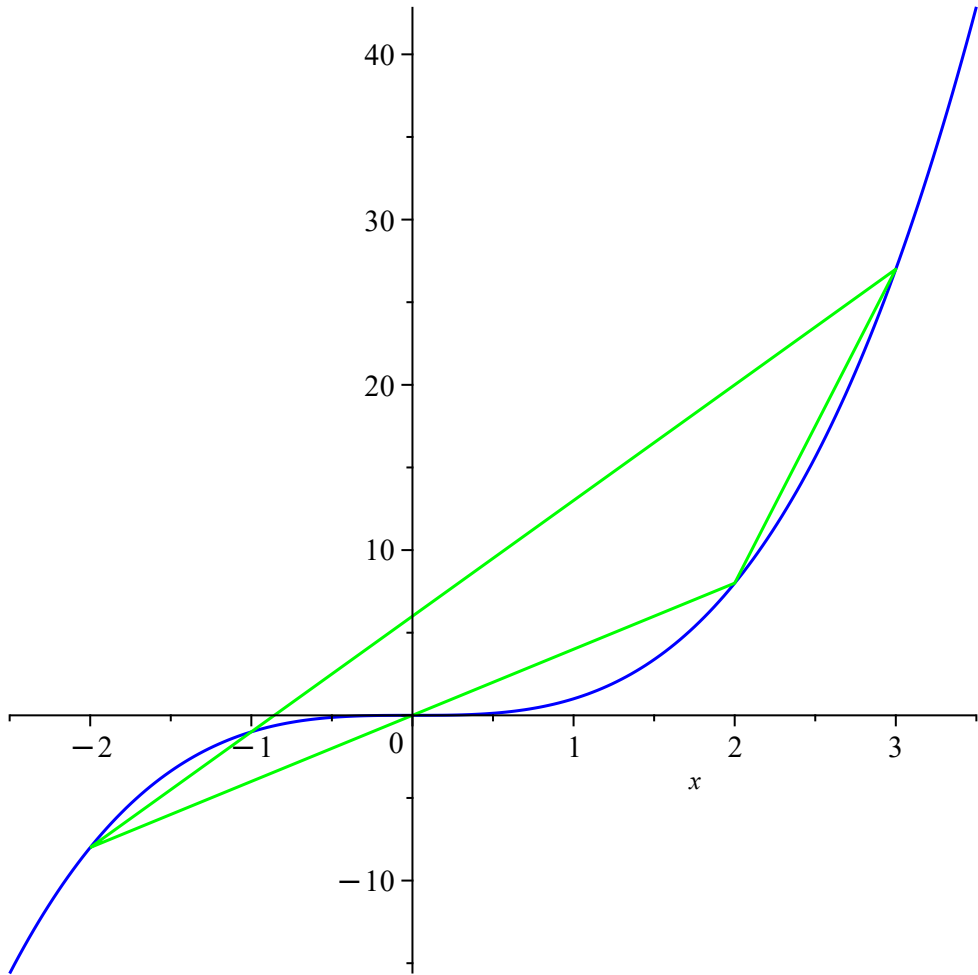
$$\text{factor} \quad - \frac{(t_2 - t_3) (t_1 - t_3) (t_1 - t_2) (t_1 + t_3 + t_2)}{2} \quad (12)$$

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x1 := -2.5; x2 := 3.5;
t1 := -2; t2 := 2; t3 := 3;
iplt := plot(y(x), x = x1..x2, colour = blue) :
trplt := plot([A1, A2, A3, A1], colour = green, ) :
plots := display(iplt, trplt);
unassign('t1','t2','t3');

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$$\begin{aligned}x1 &:= -2.5 \\ x2 &:= 3.5 \\ t_1 &:= -2 \\ t_2 &:= 2 \\ t_3 &:= 3\end{aligned}$$



$$\begin{aligned}fn(x,y) &:= x^2 + y^2 = 1 \\ fn &:= (x,y) \quad y^2 + x^2 = 1\end{aligned}\tag{13}$$

$$\begin{aligned}pln(x,t) &:= t \cdot x + t \\ pln &:= (x,t) \quad t \cdot x + t\end{aligned}\tag{14}$$

$$\begin{aligned}A1 &:= \left[\frac{1-t_1^2}{1+t_1^2}, \frac{2t_1}{1+t_1^2} \right] \\ A1 &:= \left[\frac{-t_1^2+1}{t_1^2+1}, \frac{2t_1}{t_1^2+1} \right]\end{aligned}\tag{15}$$

$$\begin{aligned}A2 &:= \left[\frac{1-t_2^2}{1+t_2^2}, \frac{2t_2}{1+t_2^2} \right] \\ A2 &:= \left[\frac{-t_2^2+1}{t_2^2+1}, \frac{2t_2}{t_2^2+1} \right]\end{aligned}\tag{16}$$

$$A3 := \left[\frac{1-t_3^2}{1+t_3^2}, \frac{2t_3}{1+t_3^2} \right]$$

$$A3 := \left[\frac{-t_3^2 + 1}{t_3^2 + 1}, \frac{2t_3}{t_3^2 + 1} \right] \quad (17)$$

$$sA1A2A3 := \text{SignedArea}(A1, A2) + \text{SignedArea}(A2, A3) + \text{SignedArea}(A3, A1)$$

$$sA1A2A3 := \frac{\left(-t_1^2 + 1\right)t_2}{\left(t_1^2 + 1\right)\left(t_2^2 + 1\right)} - \frac{t_1\left(-t_2^2 + 1\right)}{\left(t_1^2 + 1\right)\left(t_2^2 + 1\right)} + \frac{\left(-t_2^2 + 1\right)t_3}{\left(t_2^2 + 1\right)\left(t_3^2 + 1\right)} - \frac{t_2\left(-t_3^2 + 1\right)}{\left(t_2^2 + 1\right)\left(t_3^2 + 1\right)} \quad (18)$$

$$+ \frac{\left(-t_3^2 + 1\right)t_1}{\left(t_3^2 + 1\right)\left(t_1^2 + 1\right)} - \frac{t_3\left(-t_1^2 + 1\right)}{\left(t_3^2 + 1\right)\left(t_1^2 + 1\right)}$$

factor

$$- \frac{2\left(t_2 - t_3\right)\left(t_1 - t_3\right)\left(t_1 - t_2\right)}{\left(t_1^2 + 1\right)\left(t_2^2 + 1\right)\left(t_3^2 + 1\right)} \quad (19)$$

$$x1 := -1.5; x2 := 1.5;$$

$$t_1 := -8; t_2 := \frac{3}{4}; t_3 := -.5;$$

$$lplt := \text{plot}\left(\text{pln}(x, t_2), x = -1.5..1.5, \text{colour} = \text{black}\right) :$$

$$iplt := \text{plots:-implicitplot}\left(\text{fn}(x, y), x = x1..x2, y = -1.5..1.5, \text{colour} = \text{blue}, \text{scaling} = \text{constrained}\right) :$$

$$trplt := \text{plot}\left([A1, A2, A3, A1], \text{colour} = \text{green}, \right) :$$

$$\text{plots} : - \text{display}(iplt, trplt, lplt);$$

$$\text{unassign}\left('t_1', 't_2', 't_3'\right);$$

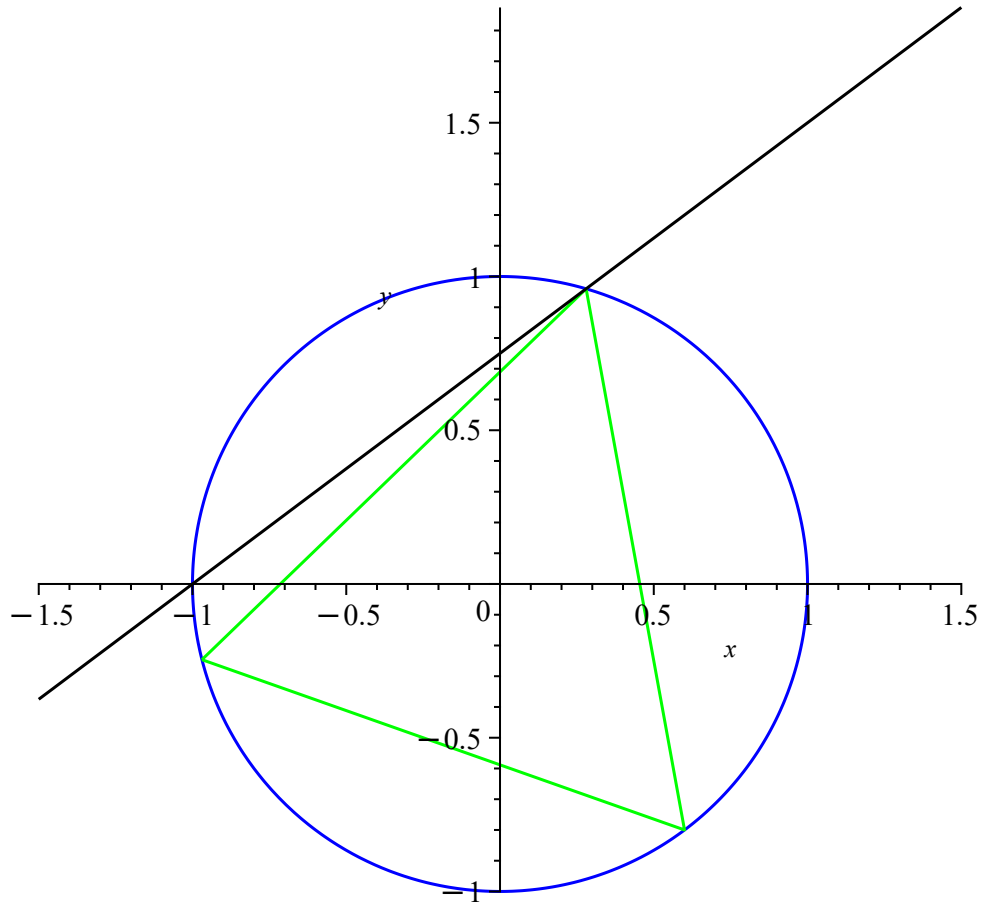
$$x1 := -1.5$$

$$x2 := 1.5$$

$$t_1 := -8$$

$$t_2 := \frac{3}{4}$$

$$t_3 := -0.5$$



$$y := (x) \quad \frac{1}{x}$$

$$y := x \quad \frac{1}{x} \tag{20}$$

$$A1 := [t_1, y(t_1)]$$

$$A1 := \left[t_1, \frac{1}{t_1} \right] \tag{21}$$

$$A2 := [t_2, y(t_2)]$$

$$A2 := \left[t_2, \frac{1}{t_2} \right] \tag{22}$$

$$A3 := [t_3, y(t_3)]$$

$$A3 := \left[t_3, \frac{1}{t_3} \right] \tag{23}$$

$$sA1A2A3 := \text{SignedArea}(A1, A2) + \text{SignedArea}(A2, A3) + \text{SignedArea}(A3, A1)$$

$$sA1A2A3 := \frac{t_1}{2t_2} - \frac{t_2}{2t_1} + \frac{t_2}{2t_3} - \frac{t_3}{2t_2} + \frac{t_3}{2t_1} - \frac{t_1}{2t_3} \tag{24}$$

factor

$$- \frac{(t_2 - t_3)(t_1 - t_3)(t_1 - t_2)}{2t_2t_1t_3} \tag{25}$$

$$x1 := -2; x2 := 2;$$

$$t_1 := -\frac{1}{2}; t_2 := \frac{1}{4}; t_3 := 1.5;$$

$$iplt := \text{plot}(y(x), x = x1..x2, \text{colour} = \text{blue}) :$$

$$trplt := \text{plot}([A1, A2, A3, A1], \text{colour} = \text{green},) :$$

$$\text{plots} := \text{display}(iplt, trplt);$$

$$\text{unassign}('t_1', 't_2', 't_3');$$

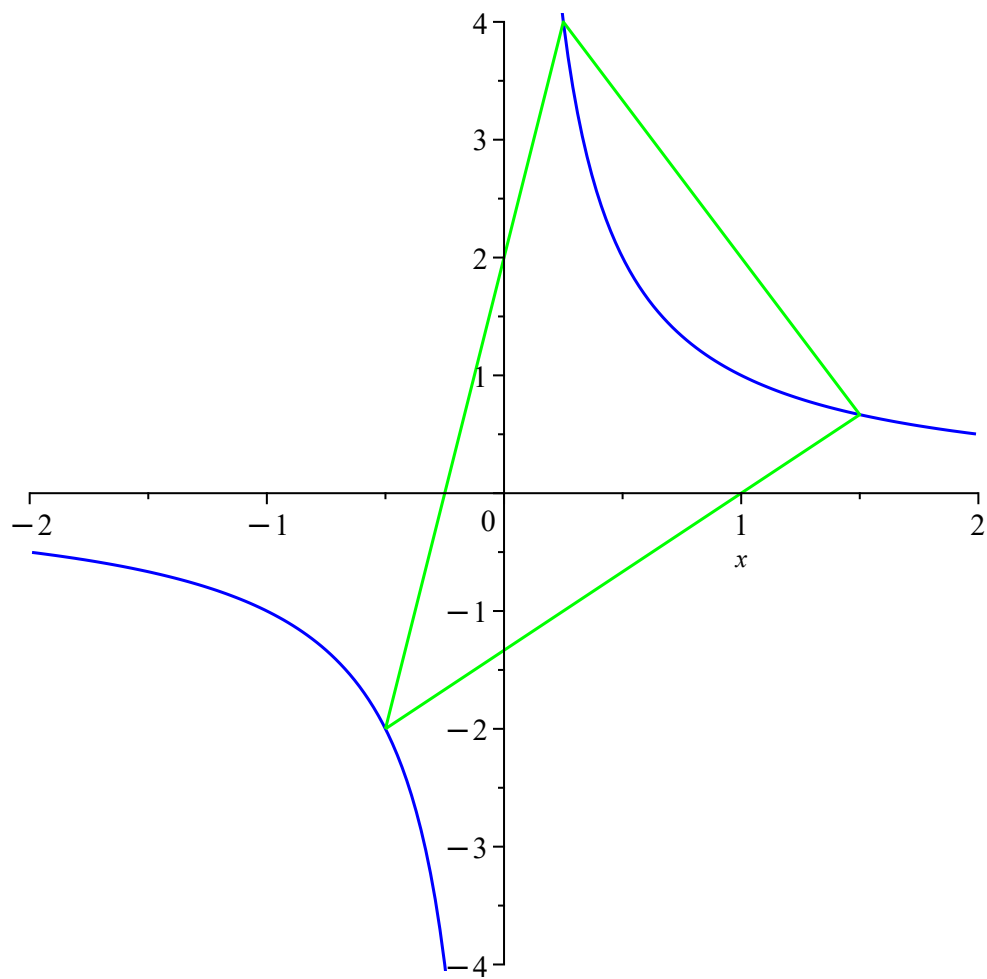
$$x1 := -2$$

$$x2 := 2$$

$$t_1 := -\frac{1}{2}$$

$$t_2 := \frac{1}{4}$$

$$t_3 := 1.5$$



$$fn(x, y) := x^2 - y^2 = 1$$

$$fn := (x, y) \quad x^2 - y^2 = 1 \quad (26)$$

$$pln(x, t) := t \cdot x + t$$

$$pln := (x, t) \quad t \cdot x + t \quad (27)$$

$$A1 := \left[\frac{1 + t_1^2}{1 - t_1^2}, \frac{2t_1}{1 - t_1^2} \right]$$

$$A1 := \left[\frac{t_1^2 + 1}{-t_1^2 + 1}, \frac{2t_1}{-t_1^2 + 1} \right] \quad (28)$$

$$A2 := \left[\frac{1 + t_2^2}{1 - t_2^2}, \frac{2t_2}{1 - t_2^2} \right]$$

$$A2 := \left[\frac{t_2^2 + 1}{-t_2^2 + 1}, \frac{2t_2}{-t_2^2 + 1} \right] \quad (29)$$

$$A3 := \left[\frac{1 + t_3^2}{1 - t_3^2}, \frac{2t_3}{1 - t_3^2} \right]$$

$$A3 := \left[\frac{t_3^2 + 1}{-t_3^2 + 1}, \frac{2t_3}{-t_3^2 + 1} \right] \quad (30)$$

$$sA1A2A3 := \text{SignedArea}(A1, A2) + \text{SignedArea}(A2, A3) + \text{SignedArea}(A3, A1)$$

$$sA1A2A3 := \frac{\left(t_1^2 + 1\right)t_2}{\left(-t_1^2 + 1\right)\left(-t_2^2 + 1\right)} - \frac{t_1\left(t_2^2 + 1\right)}{\left(-t_1^2 + 1\right)\left(-t_2^2 + 1\right)} + \frac{\left(t_2^2 + 1\right)t_3}{\left(-t_2^2 + 1\right)\left(-t_3^2 + 1\right)} \quad (31)$$

$$- \frac{t_2\left(t_3^2 + 1\right)}{\left(-t_2^2 + 1\right)\left(-t_3^2 + 1\right)} + \frac{\left(t_3^2 + 1\right)t_1}{\left(-t_3^2 + 1\right)\left(-t_1^2 + 1\right)} - \frac{t_3\left(t_1^2 + 1\right)}{\left(-t_3^2 + 1\right)\left(-t_1^2 + 1\right)}$$

factor

$$- \frac{2\left(t_2 - t_3\right)\left(t_1 - t_3\right)\left(t_1 - t_2\right)}{\left(t_1 - 1\right)\left(t_1 + 1\right)\left(t_2 - 1\right)\left(t_2 + 1\right)\left(t_3 - 1\right)\left(t_3 + 1\right)} \quad (32)$$

simplify

$$- \frac{2\left(t_2 - t_3\right)\left(t_1 - t_3\right)\left(t_1 - t_2\right)}{\left(t_1^2 - 1\right)\left(t_2^2 - 1\right)\left(t_3^2 - 1\right)} \quad (33)$$

```

x1 := -3.5; x2 := 3.5;
t1 := -3; t2 := 1/4; t3 := -.5;
lplt := plot(pln(x, t2), x = x1..x2, colour = black) :
asym := plot({x, -x}, x = x1..x2, colour = red) :
iplt := plots:-implicitplot(fn(x, y), x = x1..x2, y = -3.5..3.5, colour = blue, scaling = constrained) :
trplt := plot([A1, A2, A3, A1], colour = green, ) :
plots :- display(iplt, trplt, lplt, asym);
unassign('t1','t2','t3');

```

x1 := -3.5

x2 := 3.5

t1 := -3

t2 := 1/4

t3 := -0.5

