Helicopter Math Model

7/15/2022

$tm_0 := \text{time}(0)$

Basic Parameters

Blades are rectangular, there are $N_{bld} := 4$ of them

$D_{rtr} := 30\ ft$ $\quad R_{tip} := \frac{D_{rtr}}{2} = 15\ ft$ $\quad R_{hng} := 1\ ft$ $\quad chd := 8\ in$

$\rho_{air} := 0.076\ \frac{lb}{ft^3}$ $\quad a_{air} := 1116.324\ \frac{ft}{sec}$ $\quad$ solidity is $\sigma := \frac{N_{bld} \cdot chd}{\pi \cdot R_{tip}} = 0.057$

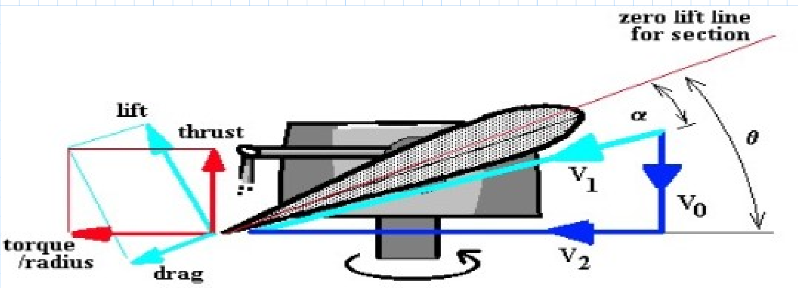
If the tip Mach Number is $M_{tip} := 0.62$, then the angular velocity is $\omega := \frac{M_{tip} \cdot a_{air}}{R_{tip}} = 440.618\ rpm$

$tm_1 := \text{time}(0)$

NACA 0012 lift and drag

$tm_2 := \text{time}(0)$

Physics and Discusion



$V_2 = \omega \cdot r$

$V_0 = u$

$\alpha = atan\left(\frac{u}{\omega \cdot r}\right)$

The lift and drag on an airfoil depend on three factors:

- the angle of the airfoil to the local velocity vector (angle of attack)
- the magnitude of the velocity $\quad Lift = C_L(\alpha) \cdot \frac{\rho \cdot V^2}{2} \cdot Area$
- the density of the air

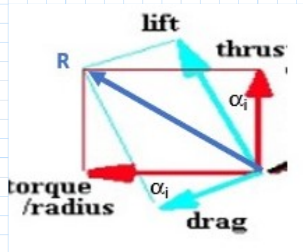
The velocity of the air at any particular radius is the linear speed $(\omega \cdot r)$ at that radius and the downflow velocity. Because the inboard blade's linear speed is less we will twist the blade to raise the angle of attack. Twist the blade eight degrees from root to tip. The angle of the section at 75% radius is defined as the collective angle θ. $\alpha(r, \theta, u) := \theta - \text{atan}\left(\frac{u}{\omega \cdot r}\right) - 8\ deg \cdot \frac{r}{R_{tip}} + 6\ deg$

Lift and drag coefficients of a NACA0012 airfoil were developed above.

$$dl(r, \theta, u) := C_L(\alpha(r, \theta, u)) \cdot \frac{\rho_{air} \cdot \left((\omega \cdot r)^2 + u^2\right)}{2} \cdot chd \qquad dl\left(10\ ft, 10\ deg, 5\ \frac{ft}{sec}\right) = 168.424\ \frac{lbf}{ft}$$

$$dd(r, \theta, u) := C_D(\alpha(r, \theta, u)) \cdot \frac{\rho_{air} \cdot \left((\omega \cdot r)^2 + u^2\right)}{2} \cdot chd \qquad dd\left(10\ ft, 10\ deg, 5\ \frac{ft}{sec}\right) = 1.885\ \frac{lbf}{ft}$$

Lift and drag forces on an airfoil are oriented to the local velocity. Thrust and torque of a helicopter rotor are oriented to the rotor shaft.



$$Rot(a) := \begin{bmatrix} \cos(a) & -\sin(a) \\ \sin(a) & \cos(a) \end{bmatrix}$$

$$dt(r,\theta,u) := dl(r,\theta,u)\cdot\cos(\alpha(r,\theta,u)) - dd(r,\theta,u)\cdot\sin(\alpha(r,\theta,u))$$

$$dq(r,\theta,u) := dd(r,\theta,u)\cdot\cos(\alpha(r,\theta,u)) + dl(r,\theta,u)\cdot\sin(\alpha(r,\theta,u))$$

$$dt\left(10\ ft, 10\ deg, 5\ \frac{ft}{sec}\right) = 165.513\ \frac{lbf}{ft}$$

$$dq\left(10\ ft, 10\ deg, 5\ \frac{ft}{sec}\right) = 31.235\ \frac{lbf}{ft}$$

$tm_3 := \text{time}(0)$

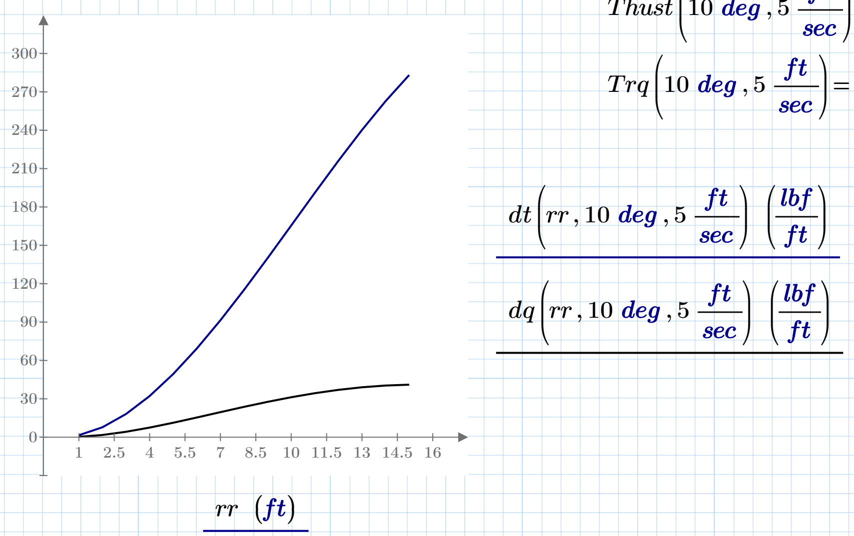
Rotor thrust and torque

$$Thust(\theta,u) := N_{bld}\cdot\int_{R_{hng}}^{R_{tip}} dt(r,\theta,u)\,dr \qquad Trq(\theta,u) := N_{bld}\cdot\int_{R_{hng}}^{R_{tip}} r\cdot dq(r,\theta,u)\,dr$$

$tm_4 := \text{time}(0)$ $\qquad rr := R_{hng}, R_{hng}+1\ ft..R_{tip}$

$$Thust\left(10\ deg, 5\ \frac{ft}{sec}\right) = 6961.196\ lbf$$

$$Trq\left(10\ deg, 5\ \frac{ft}{sec}\right) = 13131.264\ ft\cdot lbf$$



$tm_5 := \text{time}(0)$

Downflow

The rotor when it thrusts pulls air through the rotor, creating downflow (u). We can use the force balance of the thrust and the dynamic pressure to solve for that downflow. We know that

$Thrust = \frac{\rho_{air}\cdot(2\cdot u)^2}{2}\cdot\pi\cdot R_{tip}^{\ 2}$, so we can find the correct value of the downflow:

$$U(\theta) := \textbf{root}\left(Thust(\theta,u) - 2\cdot\rho_{air}\cdot u^2\cdot\pi\cdot R_{tip}^{\ 2}, u, 0\ \frac{ft}{sec}, 100\ \frac{ft}{sec}\right)$$

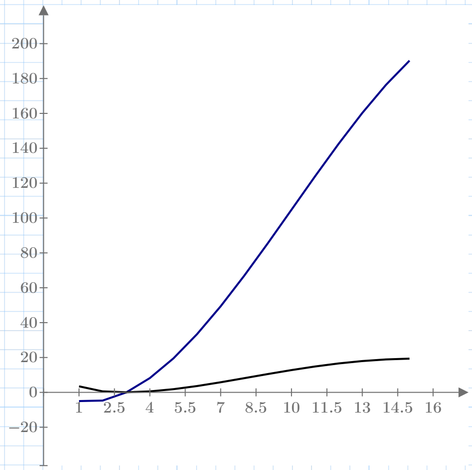
$tm_6 := \text{time}(0)$

$U(10\ deg) = 35.588\ \frac{ft}{sec}$

$Thust(10\ deg, U(10\ deg)) = 4229.333\ lbf$

$Trq(10\ deg, U(10\ deg)) = 5470.245\ ft \cdot lbf$

$\omega \cdot Trq(10\ deg, U($



$dt(rr, 10\ deg, U(10\ deg))\ \left(\frac{lbf}{ft}\right)$

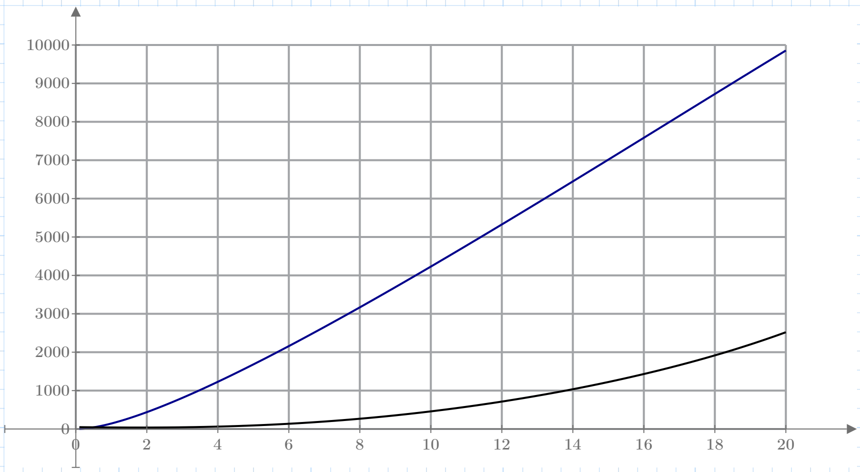
$dq(rr, 10\ deg, U(10\ deg))\ \left(\frac{lbf}{ft}\right)$

$rr\ (ft)$

$tm_7 := \text{time}(0)$

So: $THST(\theta) := Thust(\theta, U(\theta))$ $TRQ(\theta) := Trq(\theta, U(\theta))$ $qq := 0\ deg, 0.1\ deg .. 20\ deg$

$gd := Gr$



$gd^{\langle 1 \rangle}$

$THST(qq)\ (lbf)$

$\omega \cdot TRQ(qq)\ (hp)$

$gd^{\langle 0 \rangle}$

$qq\ (deg)$

$tm_8 := \text{time}(0)$

Coefficients

Thrust $C_T = \dfrac{T}{\rho \cdot \pi \cdot R_{tip}{}^4 \cdot \omega^2}$

$\rho_{air} \cdot \pi \cdot R_{tip}{}^4 \cdot \omega^2 = 799841.94\ \mathbf{lbf}$

Torque $C_Q = \dfrac{Q}{\rho \cdot \pi \cdot R_{tip}{}^5 \cdot \omega^2}$

Power $C_P = \dfrac{P}{\rho \cdot \pi \cdot R_{tip}{}^5 \cdot \omega^3}$

$C_T(\theta) := \dfrac{THST(\theta)}{\rho_{air} \cdot \pi \cdot R_{tip}{}^4 \cdot \omega^2}$

$C_Q(\theta) := \dfrac{TRQ(\theta)}{\rho_{air} \cdot \pi \cdot R_{tip}{}^5 \cdot \omega^2}$

$FM(\theta) := \dfrac{C_T(\theta)^{\frac{3}{2}}}{\sqrt{2} \cdot C_Q(\theta)}$

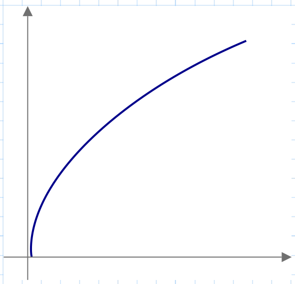
propelle

T=$C_T\rho n^2$

Q=$C_Q\rho n$

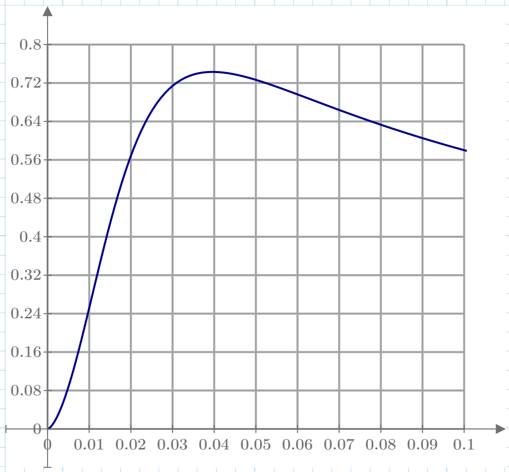
P=$C_P\rho n$

fm



$C_T(qq)$

$C_Q(qq)$



$fm^{\langle 1 \rangle}$

$FM(qq)$

$fm^{\langle 0 \rangle}$

$\dfrac{C_T(qq)}{\sigma}$

$tm_9 := \text{time}(0)$

$elps := tm - tm_0$

$elps^T = [0\ \ 0.035\ \ 0.05\ \ 0.064\ \ 0.084\ \ 0.096\ \ 0.107\ \ 0.118\ \ 0.129\ \ 0.14]$