

```
In[*]:= ClearAll["Global`*"]
```

```
In[10]:= (* ==== Parameters ==== *)
a = 988/10000000; (* m/s *)
b = -65/10000; (* m^3/m^3 *)
d = 351/1000000000; (* m^2/s *)
q = 0; (* infiltration flux at surface *)
L = 8/100; (* domain length in meters *)
theta0 = 355/1000; (* initial water content *)
thetaL = 1/10; (* lower boundary water content *)
dcoeff = d;
```

```
In[18]:= (* Constants derived from Hopf-Cole transform *)
mu = (a/dcoeff) * (theta0 + b)
c = (a/dcoeff) * (thetaL + b);
```

```
Out[18]=

$$\frac{13243}{135}$$

```

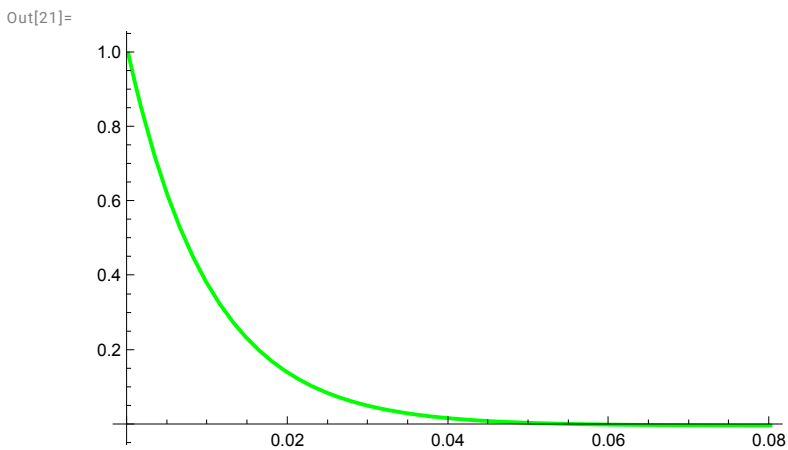
```
In[20]:= ic = w0[x_] = Exp[-mu x] (*initial condition u(x,0)*)
```

```
Out[20]=

$$e^{-13243 x/135}$$

```

```
In[21]:= Plot[Exp[-mu x], {x, 0, L}, PlotRange -> All, PlotStyle -> {Green, Thinning}]
```



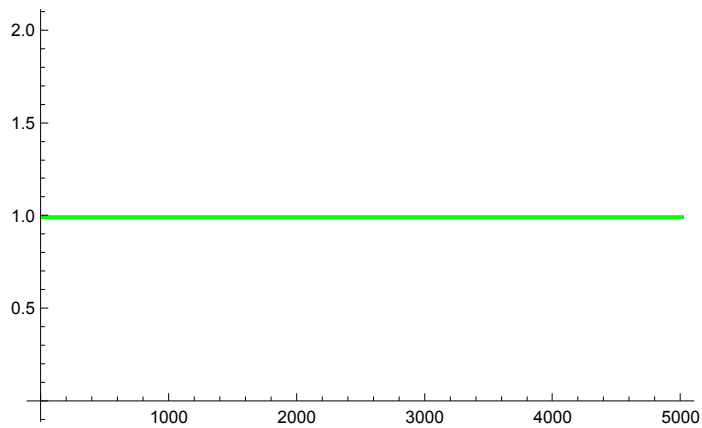
```
In[38]:= bc = g0[t_] = Exp[a * q * t / d] (*Dirichlet boundary condition u(0,t)*)
```

```
Out[38]=
1
```

```
In[39]:= (*Boundary Condititon's plot*)
```

```
Plot[Exp[a * q * t / d], {t, 0, 5000}, PlotRange -> All, PlotStyle -> {Green, Thinning}]
```

```
Out[39]=
```

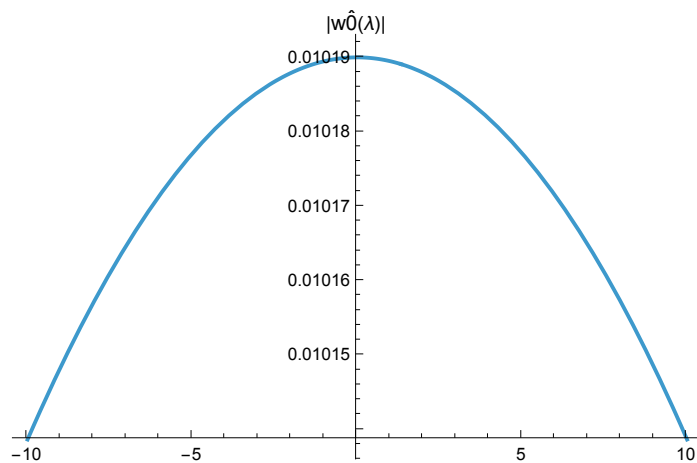


```
In[40]:= (* Fokas integral kernels *)
```

```
w0Hat[λ_] := (1 - Exp[-(μ + I λ) L]) / (μ + I λ);
```

```
In[25]:= Plot[Abs[w0Hat[λ]], {λ, -10, 10}, PlotLabel -> "|w0̂(λ)|"]
```

```
Out[25]=
```



```
In[26]:= g0Tilde[λ_, t_] :=
```

```
(Exp[(dcoeff λ^2 + (a q / dcoeff)) t] - 1) / (dcoeff λ^2 + (a q / dcoeff));
```

```
In[*]:= (*poles*)
```

```
In[27]:= λpole = I Sqrt[(a q) / dcoeff^2]
```

```
Out[27]=
```

```
0
```

$$\begin{aligned}
 \text{In[28]:= } v[\lambda_-, x_-, t_-] := & \frac{1}{2 * \text{Pi}} * \frac{\text{Exp}[-\text{dcoeff} * \lambda^2 * t]}{\lambda * \text{Cos}[\lambda L] + c * \text{Sin}[\lambda L]} * \\
 & \left(\left(\frac{\text{Exp}[I * \lambda * x] * (\mu - I * \lambda) - \text{Exp}[-\mu * L] * \text{Exp}[I * \lambda * (L - x)] * (\mu + I * \lambda)}{\mu^2 + \lambda^2} \right) * \right. \\
 & (\lambda * \text{Cos}[\lambda L] + c * \text{Sin}[\lambda L]) \\
 & - \left(\text{Exp}[I * \lambda * L] * (I * \lambda + c) * \text{Sin}[\lambda x] * \frac{1 - \text{Exp}[-\mu * L] * \text{Exp}[-I * \lambda * L]}{\mu + I * \lambda} \right) \\
 & - (\lambda * \text{Cos}[\lambda (L - x)] + c * \text{Sin}[\lambda (L - x)]) * \frac{1 - \text{Exp}[-\mu * L] * \text{Exp}[I * \lambda * L]}{\mu - I * \lambda} \\
 & \left. + 2 * I * \lambda * \text{dcoeff} * (\lambda * \text{Cos}[\lambda (L - x)] + c * \text{Sin}[\lambda (L - x)]) * \frac{1}{\text{dcoeff} * \lambda^2 + \frac{a * q}{\text{dcoeff}}} \right)
 \end{aligned}$$

In[*]:=

In[29]:= (*Define the three contours k1,k2,k3*)

l = 5;

$\lambda 1[r_] = l + I + r * \text{Exp}[I * \text{Pi} / 6]$

$\lambda 2[r_] = -l + I + r * \text{Exp}[I * 5 * \text{Pi} / 6]$

$\lambda 3[s_] = s + I$

Out[30]=

$$(5 + i) + e^{\frac{i\pi}{6}} r$$

Out[31]=

$$(-5 + i) + e^{\frac{5i\pi}{6}} r$$

Out[32]=

$$i + s$$

In[33]:= (*Derivatives of the contours*)

$d\lambda 1[r_] := D[\lambda 1[r], r];$

$d\lambda 2[r_] := D[\lambda 2[r], r];$

$d\lambda 3[s_] := D[\lambda 3[s], s];$

In[36]:= (*Define the approxU function with NumericQ guards*)

```

approxW[x_?NumericQ, t_?NumericQ] := Module[{temp1, temp2},
  temp1 = NIntegrate[Re[v[\lambda 1[r], x, t] * d\lambda 1[r] - v[\lambda 2[r], x, t] * d\lambda 2[r]],
    {r, 0, Infinity}, AccuracyGoal -> 8, PrecisionGoal -> 8, MaxRecursion -> 25];
  temp2 = NIntegrate[Re[v[\lambda 3[s], x, t] * d\lambda 3[s]], {s, -l, l},
    AccuracyGoal -> 8, PrecisionGoal -> 8, MaxRecursion -> 25];
  temp1 + temp2]

```

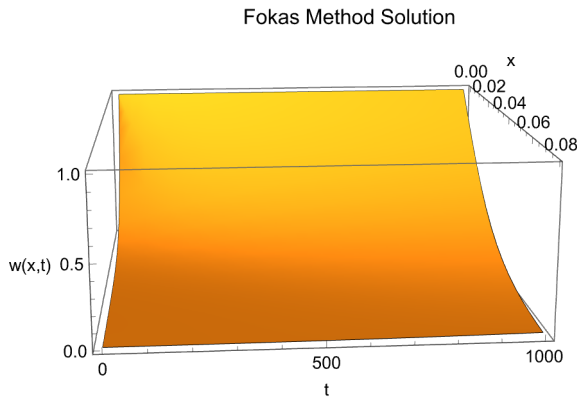
In[62]:= (*Example: Evaluate at a point*) approxW[0.01, 0.1]

Out[62]=

0.375077

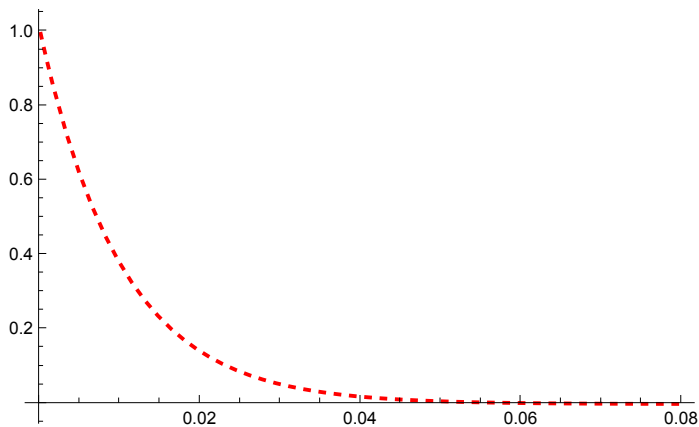
```
In[ ]:= (*Step 4:Plot the 3D surface*)
Plot3D[approxW[x, t], {x, 0, L}, {t, 0, 1000}, AxesLabel -> {"x", "t", "w(x,t)"},
PlotLabel -> "Fokas Method Solution", Mesh -> None, PlotRange -> All]
```

Out[]:=

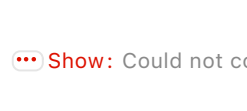


```
In[ ]:= Plot[approxW[x, 0], {x, 0, L}, PlotRange -> All, PlotStyle -> {Red, Dashed, Thick}]
```

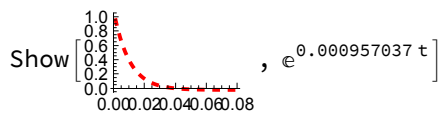
Out[]:=



```
In[ ]:= Show[%, %14]
```

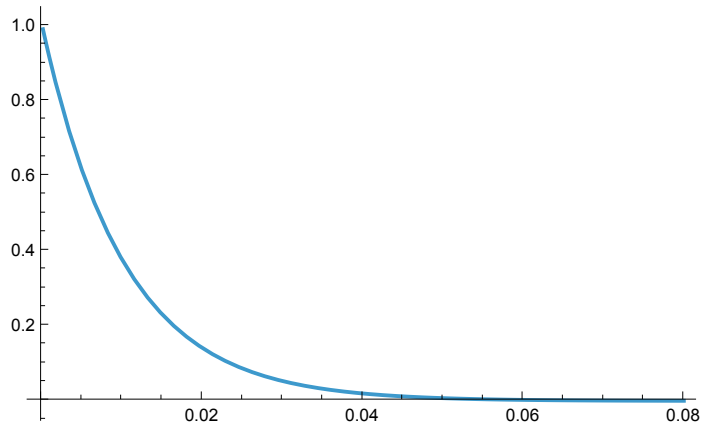
Show: Could not combine the graphics objects in Show[, $e^{0.000957037 t}$].

Out[]:=



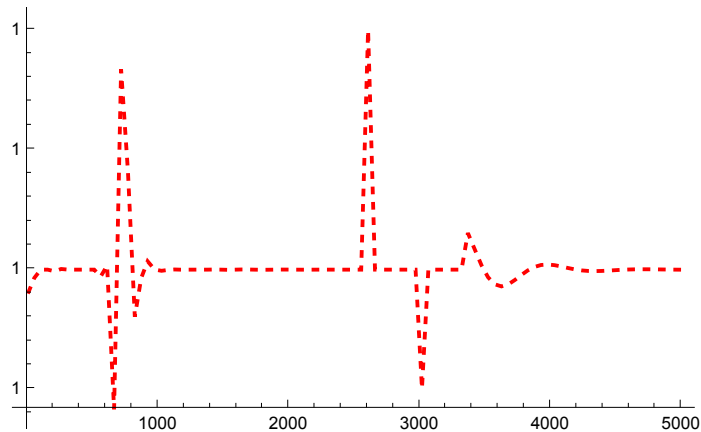
```
In[*]:= Plot[approxW[x, 0], {x, L / 1000, 999 L / 1000}, PlotRange -> All]
```

```
Out[*]=
```



```
In[*]:= Plot[approxW[0, t], {t, 0, 5000}, PlotRange -> All, PlotStyle -> {Red, Dashed, Thick}]
```

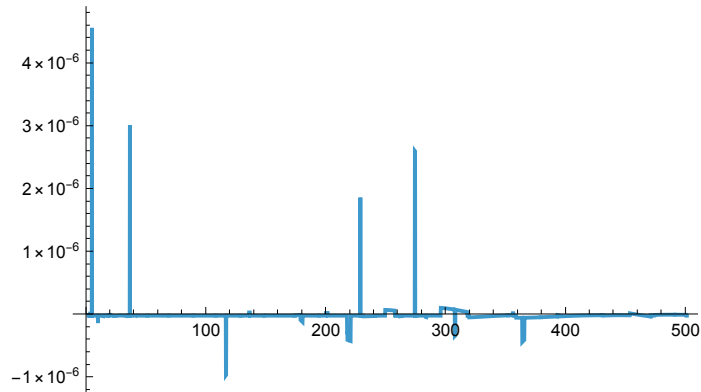
```
Out[*]=
```



```
In[41]:= (*Define the approxU function with NumericQ guards*)
dapproxW[x_?NumericQ, t_?NumericQ] := Module[{temp1, temp2},
  temp1 = NIntegrate[Re[(D[v[\lambda1[r], xx, t], xx] /. xx -> x) * d\lambda1[r] -
    (D[v[\lambda2[r], xx, t], xx] /. xx -> x) * d\lambda2[r]], {r, 0, Infinity},
    AccuracyGoal -> 8, PrecisionGoal -> 8, MaxRecursion -> 25];
  temp2 = NIntegrate[Re[(D[v[\lambda3[s], xx, t], xx] /. xx -> x) * d\lambda3[s]],
    {s, -1, 1}, AccuracyGoal -> 8, PrecisionGoal -> 8, MaxRecursion -> 25];
  temp1 + temp2]
```

```
In[42]:= Plot[(dapproxW[x, t] /. x → L) + c approxW[L, t], {t, 1, 500}, PlotRange → All]
```

```
Out[42]=
```



```
In[43]:= dapproxW[0.01, 0.1]
```

```
Out[43]=
```

```
-36.7936
```

```
In[44]:= thetaApprox[x_?NumericQ, t_?NumericQ] :=  
Re[-(dcoeff / a) (dapproxW[x, t] / approxW[x, t]) - b]
```

```
In[45]:= thetaApprox[0.01, 0.1]
```

```
Out[45]=
```

```
0.355
```

```
In[56]:= timesMin = {1, 5, 15, 30, 60, 120};
```

```
timesSec = 60 * timesMin;
```

```
xgrid = Subdivide[0, L, 50]
```

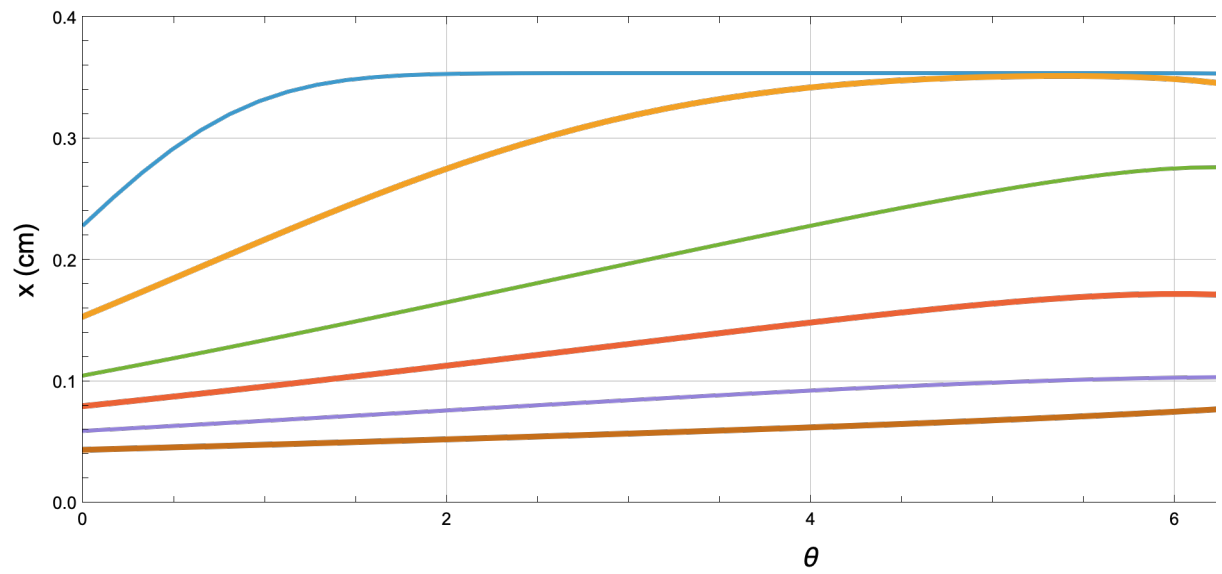
```
profiles = Table[Table[{100 x, thetaApprox[x, t]}, {x, xgrid}], {t, timesSec}];
```

```
plt = ListLinePlot[profiles, Frame → True,  
FrameLabel → {Style["θ", 14], Style["x (cm)", 14]}, PlotLegends →  
Placed[LineLegend[Table[Row[{NumberForm[t, {4, 1}], " min"}], {t, timesMin}]],  
Right], PlotStyle → {Thick, AbsoluteThickness[3]}, PlotRange →  
{0, 8}, (*θ range*){0, 0.4} (*vertical axis: 0 at top, 25 at bottom*),  
AspectRatio → 1/3, (*wider horizontal axis*) ImageSize → {800, 700},  
GridLines → {Automatic, Automatic}, FrameTicks → Automatic]
```

```
Out[58]=
```

```
{0,  $\frac{1}{625}$ ,  $\frac{2}{625}$ ,  $\frac{3}{625}$ ,  $\frac{4}{625}$ ,  $\frac{1}{125}$ ,  $\frac{6}{625}$ ,  $\frac{7}{625}$ ,  $\frac{8}{625}$ ,  $\frac{9}{625}$ ,  $\frac{2}{125}$ ,  $\frac{11}{625}$ ,  $\frac{12}{625}$ ,  
 $\frac{13}{625}$ ,  $\frac{14}{625}$ ,  $\frac{3}{125}$ ,  $\frac{16}{625}$ ,  $\frac{17}{625}$ ,  $\frac{18}{625}$ ,  $\frac{19}{625}$ ,  $\frac{4}{125}$ ,  $\frac{21}{625}$ ,  $\frac{22}{625}$ ,  $\frac{23}{625}$ ,  $\frac{24}{625}$ ,  
 $\frac{1}{25}$ ,  $\frac{26}{625}$ ,  $\frac{27}{625}$ ,  $\frac{28}{625}$ ,  $\frac{29}{625}$ ,  $\frac{6}{125}$ ,  $\frac{31}{625}$ ,  $\frac{32}{625}$ ,  $\frac{33}{625}$ ,  $\frac{34}{625}$ ,  $\frac{7}{125}$ ,  $\frac{36}{625}$ ,  $\frac{37}{625}$ ,  
 $\frac{38}{625}$ ,  $\frac{39}{625}$ ,  $\frac{8}{125}$ ,  $\frac{41}{625}$ ,  $\frac{42}{625}$ ,  $\frac{43}{625}$ ,  $\frac{44}{625}$ ,  $\frac{9}{125}$ ,  $\frac{46}{625}$ ,  $\frac{47}{625}$ ,  $\frac{48}{625}$ ,  $\frac{49}{625}$ ,  $\frac{2}{25}$ }
```

Out[60]=



```
In[*]:= Rotate[Rasterize[plt], -Pi / 2]
Out[*]=
```

```
In[*]:= Export["Users/thanassisparaskevopoulos/Desktop/PHD/Fokas
             method/First Paper/3d_fokas_solution_ex2.png", %48, "PNG"]
Out[*]=
/Users/thanassisparaskevopoulos/Desktop/PHD/Fokas
method/First Paper/3d_fokas_solution_ex2.png
```

```
In[*]:= x =.; t =.;
(*build data on the full needed time range*)
dat1 = ParallelTable[Re[-(dcoeff / a) * (dapproxW[X, T] / approxW[X, T]) - b],
                    {X, 0, L, L / 50}, {T, 0, 7200, 7200 / 50}];
```

 **ParallelTable:** No parallel kernels available; proceeding with sequential evaluation.

NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\ll 2 \gg]}(1 - \text{Power}[\ll 2 \gg])((5+i) + \text{Power}[\ll 2 \gg])r\left(\frac{3553}{135} + i\text{Plus}[\ll 2 \gg]\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \frac{(\ll 1 \gg + i\ll 2 \gg)\ll 1 \gg}{\frac{1\ll 6 \gg 49}{18225} + \ll 1 \gg} + \frac{\ll 1 \gg}{\ll 1 \gg} - \frac{(1 - \text{Power}[\ll 2 \gg])\ll 1 \gg}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\ll 2 \gg]) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 2 \gg]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\ll 1 \gg}\ll 2 \gg\left(\frac{3553}{135} + i\ll 1 \gg\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \ll 4 \gg \right)}{2\pi \left((-5+i) + \text{Times}[\ll 2 \gg] \right) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 1 \gg]]} \right]$$

has evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\ll 2 \gg]}(1 - \text{Power}[\ll 2 \gg])((5+i) + \text{Power}[\ll 2 \gg])r\left(\frac{3553}{135} + i\text{Plus}[\ll 2 \gg]\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \frac{(\ll 1 \gg + i\ll 2 \gg)\ll 1 \gg}{\frac{1\ll 6 \gg 49}{18225} + \ll 1 \gg} + \frac{\ll 1 \gg}{\ll 1 \gg} - \frac{(1 - \text{Power}[\ll 2 \gg])\ll 1 \gg}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\ll 2 \gg]) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 2 \gg]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\ll 1 \gg}\ll 2 \gg\left(\frac{3553}{135} + i\ll 1 \gg\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \ll 4 \gg \right)}{2\pi \left((-5+i) + \text{Times}[\ll 2 \gg] \right) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 1 \gg]]} \right]$$

has evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\ll 2 \gg]}(1 - \text{Power}[\ll 2 \gg])((5+i) + \text{Power}[\ll 2 \gg])r\left(\frac{3553}{135} + i\text{Plus}[\ll 2 \gg]\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \frac{(\ll 1 \gg + i\ll 2 \gg)\ll 1 \gg}{\frac{1\ll 6 \gg 49}{18225} + \ll 1 \gg} + \frac{\ll 1 \gg}{\ll 1 \gg} - \frac{(1 - \text{Power}[\ll 2 \gg])\ll 1 \gg}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\ll 2 \gg]) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 2 \gg]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\ll 1 \gg}\ll 2 \gg\left(\frac{3553}{135} + i\ll 1 \gg\right)}{\frac{13243}{135} + \text{Times}[\ll 2 \gg]} + \ll 4 \gg \right)}{2\pi \left((-5+i) + \text{Times}[\ll 2 \gg] \right) \cos[\text{Times}[\ll 2 \gg]] + \frac{3553}{135} \sin[\text{Times}[\ll 1 \gg]]} \right]$$

has evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

General: Further output of NIntegrate::inumri will be suppressed during this calculation. [i](#)

In[*]:= vals = Partition[dat1, 51];

⋯ NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\llbracket 2 \rrbracket]} (1 - \text{Power}[\llbracket 2 \rrbracket]) ((5+i) + \text{Power}[\llbracket 2 \rrbracket]) r \left(\frac{3553}{135} + i\text{Plus}[\llbracket 2 \rrbracket] \right)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} + \frac{(\llbracket 1 \rrbracket + i\llbracket 2 \rrbracket \llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{\frac{1\llbracket 6 \rrbracket 49}{18225} + \llbracket 1 \rrbracket} + \frac{\llbracket 1 \rrbracket}{\llbracket 1 \rrbracket} - \frac{(1 - \text{Power}[\llbracket 2 \rrbracket]) (\llbracket 1 \rrbracket)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\text{Times}[\llbracket 2 \rrbracket]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\llbracket 1 \rrbracket} \llbracket 2 \rrbracket} \left(\frac{3553}{135} + i\llbracket 1 \rrbracket \right) + \llbracket 4 \rrbracket \right)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} + \llbracket 4 \rrbracket \right)}{2\pi \left(((-5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\text{Times}[\llbracket 1 \rrbracket]] \right)}$$

to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

⋯ NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(\frac{7106i((-5-i) - \text{Power}[\llbracket 2 \rrbracket]) r}{135((5+i) + \text{Times}[\llbracket 2 \rrbracket])} - \frac{3553 \llbracket 1 \rrbracket \llbracket 1 \rrbracket ((\llbracket 1 \rrbracket) - \llbracket 1 \rrbracket)}{135 \left(\frac{\llbracket 5 \rrbracket}{\llbracket 3 \rrbracket} + \llbracket 1 \rrbracket \right)} - \llbracket 1 \rrbracket + \frac{(i \text{Power}[\llbracket 2 \rrbracket] \text{Plus}[\llbracket 2 \rrbracket] \text{Plus}[\llbracket 2 \rrbracket] + i\llbracket 1 \rrbracket \llbracket 1 \rrbracket \text{Plus}[\llbracket 2 \rrbracket]) (\llbracket 1 \rrbracket)}{\frac{175377049}{18225} + \text{Power}[\llbracket 2 \rrbracket]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\text{Times}[\llbracket 2 \rrbracket]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(\llbracket 5 \rrbracket + \frac{(\llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{\frac{\llbracket 9 \rrbracket}{\llbracket 5 \rrbracket} \llbracket 1 \rrbracket \llbracket 1 \rrbracket} \right)}{2\pi \left(((-5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\llbracket 1 \rrbracket] \right)}$$

evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

⋯ NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\llbracket 2 \rrbracket]} (1 - \text{Power}[\llbracket 2 \rrbracket]) ((5+i) + \text{Power}[\llbracket 2 \rrbracket]) r \left(\frac{3553}{135} + i\text{Plus}[\llbracket 2 \rrbracket] \right)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} + \frac{(\llbracket 1 \rrbracket + i\llbracket 2 \rrbracket \llbracket 1 \rrbracket) \llbracket 1 \rrbracket}{\frac{1\llbracket 6 \rrbracket 49}{18225} + \llbracket 1 \rrbracket} + \frac{\llbracket 1 \rrbracket}{\llbracket 1 \rrbracket} - \frac{(1 - \text{Power}[\llbracket 2 \rrbracket]) (\llbracket 1 \rrbracket)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} \right) \right] / \left(2\pi \left(((5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\text{Times}[\llbracket 2 \rrbracket]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\llbracket 1 \rrbracket} \llbracket 2 \rrbracket} \left(\frac{3553}{135} + i\llbracket 1 \rrbracket \right) + \llbracket 4 \rrbracket \right)}{\frac{13243}{135} + \text{Times}[\llbracket 2 \rrbracket]} + \llbracket 4 \rrbracket \right)}{2\pi \left(((-5+i) + \text{Times}[\llbracket 2 \rrbracket]) \cos[\text{Times}[\llbracket 2 \rrbracket]] + \frac{3553}{135} \sin[\text{Times}[\llbracket 1 \rrbracket]] \right)}$$

to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

⋯ General: Further output of NIntegrate::inumri will be suppressed during this calculation. i

`In[*]:= ListPlot3D[vals, DataRange -> {{0, L}, {0, 7200 / 60}},
 AxesLabel -> {"x (cm)", "t (min)", "θ(x,t)"}]`

NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\langle\langle 2 \rangle\rangle]} (1 - \text{Power}[\langle\langle 2 \rangle\rangle]) ((5 + i) + \text{Power}[\langle\langle 2 \rangle\rangle] r) \left(\frac{3553}{135} + i\text{Plus}[\langle\langle 2 \rangle\rangle]\right)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} + \frac{(\langle\langle 1 \rangle\rangle + i\langle\langle 2 \rangle\rangle \langle\langle 1 \rangle\rangle) \langle\langle 1 \rangle\rangle}{\frac{1\langle\langle 6 \rangle\rangle 49}{18225} + \langle\langle 1 \rangle\rangle} + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} - \frac{(1 - \text{Power}[\langle\langle 2 \rangle\rangle]) (\langle\langle 1 \rangle\rangle)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} \right) \Bigg] / \left(2\pi \left(((5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\text{Times}[\langle\langle 2 \rangle\rangle]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\langle\langle 1 \rangle\rangle \langle\langle 2 \rangle\rangle} \left(\frac{3553}{135} + i\langle\langle 1 \rangle\rangle\right)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} + \langle\langle 4 \rangle\rangle \right)}{2\pi \left(((-5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\text{Times}[\langle\langle 1 \rangle\rangle]] \right)}$$

has evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(\frac{7106 i ((-5 - i) - \text{Power}[\langle\langle 2 \rangle\rangle] r) - \frac{3553 \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle ((\langle\langle 1 \rangle\rangle) - \langle\langle 1 \rangle\rangle)}{135 ((5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle])} - \langle\langle 1 \rangle\rangle + \frac{(i \text{Power}[\langle\langle 2 \rangle\rangle] \text{Plus}[\langle\langle 2 \rangle\rangle] \text{Plus}[\langle\langle 2 \rangle\rangle] + i \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle \text{Plus}[\langle\langle 2 \rangle\rangle]) (\langle\langle 1 \rangle\rangle)}{\frac{175377049}{18225} + \text{Power}[\langle\langle 2 \rangle\rangle]} \right) \Bigg] / \left(2\pi \left(((5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\text{Times}[\langle\langle 2 \rangle\rangle]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(\langle\langle 5 \rangle\rangle + \frac{(\langle\langle 1 \rangle\rangle) \langle\langle 1 \rangle\rangle}{\frac{\langle\langle 9 \rangle\rangle}{\langle\langle 5 \rangle\rangle} \langle\langle 1 \rangle\rangle \langle\langle 1 \rangle\rangle} \right)}{2\pi \left(((-5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\langle\langle 1 \rangle\rangle] \right)}$$

evaluated to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

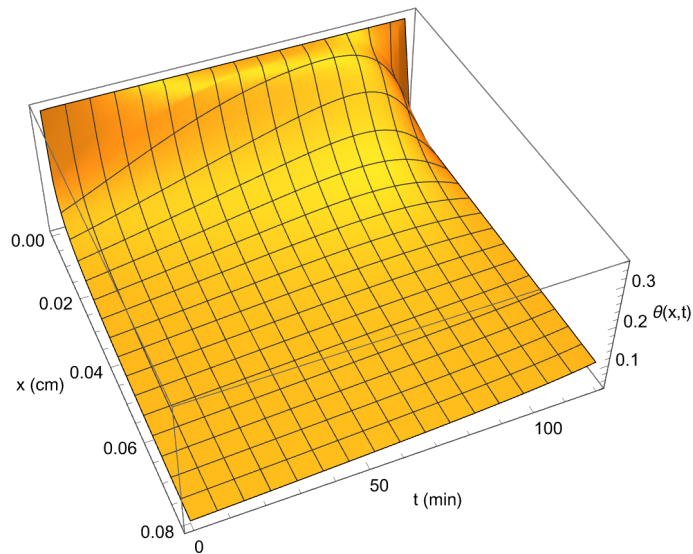
NIntegrate: The integrand

$$\operatorname{Re}\left[e^{\frac{i\pi}{6}} \left(-\frac{e^{\frac{2}{25}i\text{Plus}[\langle\langle 2 \rangle\rangle]} (1 - \text{Power}[\langle\langle 2 \rangle\rangle]) ((5 + i) + \text{Power}[\langle\langle 2 \rangle\rangle] r) \left(\frac{3553}{135} + i\text{Plus}[\langle\langle 2 \rangle\rangle]\right)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} + \frac{(\langle\langle 1 \rangle\rangle + i\langle\langle 2 \rangle\rangle \langle\langle 1 \rangle\rangle) \langle\langle 1 \rangle\rangle}{\frac{1\langle\langle 6 \rangle\rangle 49}{18225} + \langle\langle 1 \rangle\rangle} + \frac{\langle\langle 1 \rangle\rangle}{\langle\langle 1 \rangle\rangle} - \frac{(1 - \text{Power}[\langle\langle 2 \rangle\rangle]) (\langle\langle 1 \rangle\rangle)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} \right) \Bigg] / \left(2\pi \left(((5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\text{Times}[\langle\langle 2 \rangle\rangle]] \right) \right) - \frac{e^{\frac{5i\pi}{6}} \left(-\frac{e^{\langle\langle 1 \rangle\rangle \langle\langle 2 \rangle\rangle} \left(\frac{3553}{135} + i\langle\langle 1 \rangle\rangle\right)}{\frac{13243}{135} + \text{Times}[\langle\langle 2 \rangle\rangle]} + \langle\langle 4 \rangle\rangle \right)}{2\pi \left(((-5 + i) + \text{Times}[\langle\langle 2 \rangle\rangle]) \cos[\text{Times}[\langle\langle 2 \rangle\rangle]] + \frac{3553}{135} \sin[\text{Times}[\langle\langle 1 \rangle\rangle]] \right)}$$

to Overflow, Indeterminate or Infinity for all sampling points in the region with boundaries $\{0., 4.64782 \times 10^{14}\}$.

General: Further output of NIntegrate::inumri will be suppressed during this calculation. [i](#)

Out[]=



In[]:= `Export["/Users/thanassisparaskevopoulos/Desktop/3d_fokas_ex2.png", %103, "PNG"]`

Out[]=

`/Users/thanassisparaskevopoulos/Desktop/3d_fokas_ex2.png`

In[]:= `Export["/Users/thanassisparaskevopoulos/Desktop/Untitled.png", %49, "PNG"]`

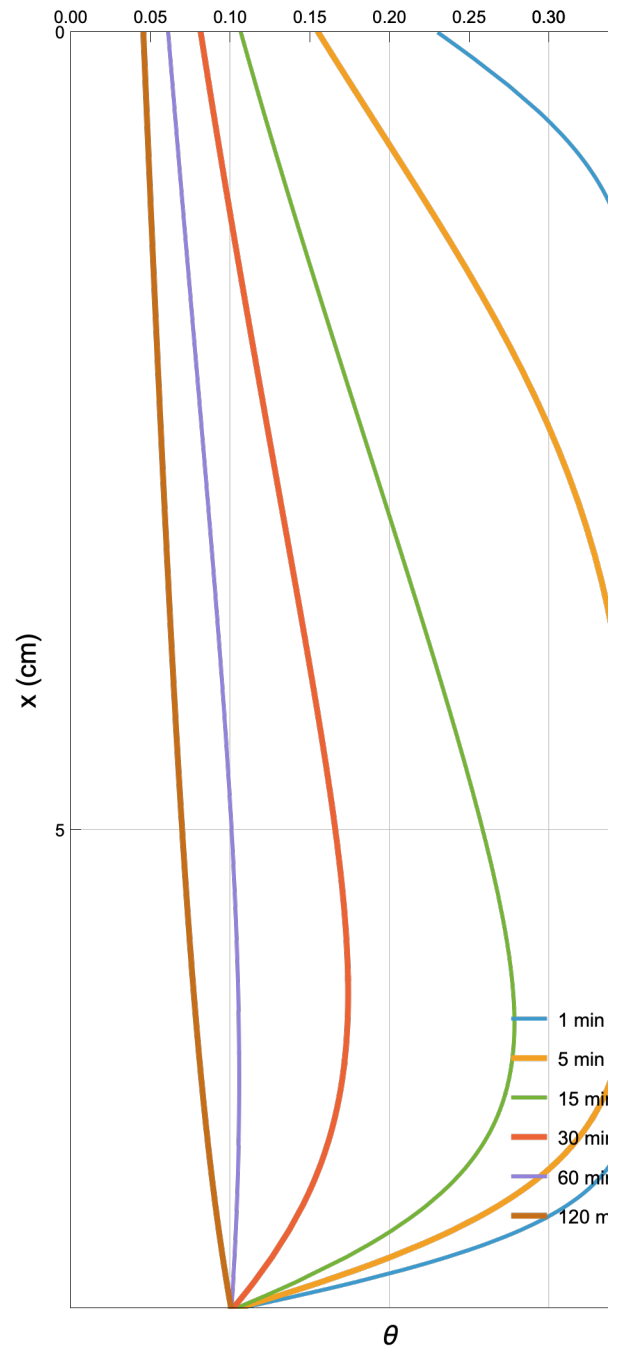
Out[]=

`/Users/thanassisparaskevopoulos/Desktop/Untitled.png`

In[]:= `topTicks = Union[Table[x, {x, 0, 0.3, 0.05}], {0.3}];`

```
pltRot = ParametricPlot[
  Evaluate@Table[{thetaApprox[x, t], -100 x}, {t, timesSec}], {x, 0, L},
  Frame → True, Axes → False, FrameLabel → {Style["θ", 14], Style["x (cm)", 14]},
  FrameTicks → {{Table[{y, Abs[y]}, {y, -25, 0, 5}], (*LEFT y-axis*)
    None (*no right y-axis*)}, {None, (*no bottom x-axis*)
    Table[{x, NumberForm[x, {3, 2}]}, {x, topTicks]} (*TOP x-axis labels*)}},
  PlotLegends → Placed[LineLegend[Table[Row[{NumberForm[t, {4, 1}], " min"}],
    {t, timesMin}]], Scaled[{0.78, 0.15}] (*legend shifted slightly left*)],
  PlotStyle → {Thick, AbsoluteThickness[3]}, PlotRange →
  {{0, 0.4}, (*θ range*){-8, 0} (*vertical axis:0 at top,25 at bottom*)},
  AspectRatio → 2, (*smaller→wider x-axis*)ImageSize → {1000, 700},
  (*optional:wider plot*)GridLines → {Automatic, Range[-25, 0, 5]}
```

Out[]=



In[*]:= Rotate[Rasterize[pltRot]]

Out[*]=

